國立成功大學

110學年度碩士班招生考試試題

編　號：42

系　所：光電科學與工程學系

科　目：工程數學

日　期：0203

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) A uniform rope is coiled loosely on the ground. One end of the rope is pulled vertically upward by means of a constant force of 5 lb. The rope weighs 1 lb per feet. Determine the height x(t) of the end above ground level at t < 1 sec.

2. (15%) Solve the wave equation subject to the given conditions.

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0,\ t > 0$$

$$u(x,\ 0) = x,\ \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0,\ 0 < x < L$$

3. (15%) If there is a heat transfer from the lateral surface of a thin wire of length L into a medium at constant temperature u_m, then the heat equation takes on the form

$$k\frac{\partial^2 u}{\partial x^2} - h(u - u_m) = \frac{\partial u}{\partial t},\ 0 < x < L,\ t > 0,$$

where h is constant. Use the **Laplace transform** to find the temperature $u(x, t)$. The initial temperature is a constant u_0 throughout and the end $x = 0$ and $x = L$ are insulated.

4. (15%) Please find the eigenvalues of the matrices A and B

$$A = \begin{bmatrix} 3 & -1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & -1 & 1 & 2 & 6 & 1 \\ 1 & 1 & -1 & 6 & 7 & 3 \\ 0 & 0 & 2 & 1 & 3 & 9 \\ 0 & 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

5. (15%) If now we have the position vector $\vec{R} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$ and the spatial distance $r = \sqrt{x^2 + y^2 + z^2} \neq 0$, please find (a) $\nabla(r^n)$ for $n = 1, 2$, k。(b) the condition with which $\nabla^2 r^n = 0$。(c) the value of n so that $\nabla \cdot (r^n \vec{R}) = 0$。

6. (20%) If $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x < 0 \end{cases}$, please (a) find the Fourier integral of $f(x)$。(b) calculate $\int_0^\infty \frac{\cos x}{1+x^2} dx$ using the result obtained from (a)。