國立成功大學

111學年度碩士班招生考試試題

編　號：40

系　所：光電科學與工程學系

科　目：近代物理

日　期：0220

節　次：第 1 節

備　註：不可使用計算機

編號： 40 國立成功大學 111 學年度碩士班招生考試試題

系　　所：光電科學與工程學系

考試科目：近代物理　　考試日期：0220，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (8%) If two photons are generated when an electron-positron annihilation occurs at rest, please find the energy and the momentum of the annihilation photons.

2. Consider a potential well $U(x) = \begin{cases} 0 & 0 \le x \le a \\ \infty & x < 0 \; and \; x > a \end{cases}$.

An electron (mass = m) with the wavefunction $\Psi(x,0) = C_0\psi_0 + \frac{1}{\sqrt{3}}\psi_1$ is trapped in above potential well (ψ_0 and ψ_1 are both normalized), please:

(a) Find C_0 (3%)

(b) What are the potentially-detectable energies of the electron at $t = 0$? (3%)

(c) Find the expectation value of the electron's energy at $t = 0$ (3%)

(d) Find the wavefunction of the electron at t, i.e., $\Psi(x, t)$ (3%)

(e) Calculate the expectation value of the electron's energy at $t \neq 0$ (3%)

3. There are five electrons placed in a box. If the volume of the box $V = a^3$, please find:

(a) The energy of the ground state and the first excited state (8%)

(b) The number of the degenerate energy level for the ground state and the first excited state (8%)

4. (5%) Please try to verify the uncertainty principle $\Delta x \Delta p \geqq \hbar/2$ with the electron diffraction experiment (the width of the slit is a).

5. For a hydrogen atom $\Psi_{100} = Ce^{-r/a}$, please

(a) (3%) Determine the normalization constant C. (not that $\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}$)

(b) (3%) Find the radial probability density P(r)

6. In general, an electron in an atom has both orbital angular momentum characterized by the quantum number l and spin angular momentum characterized by the quantum number s.
   (a) Find the total angular momentum quantum number j. (3%)
   (b) Find the z component of total angular momentum $\vec{J}$. (3%)
   (c) If $\vec{J}_1$ is one angular momentum (orbital, spin, or a combination) and $\vec{J}_2$ is another, the resulting total angular momentum is $\vec{J} = \vec{J}_1 + \vec{J}_2$. Find the possible values of total angular momentum quantum number j. (3%)
   (d) Two electrons each have zero orbital angular momentum. What are the possible quantum numbers for the total angular momentum of the two-electron system? (3%)

(e) An electron in an atom has orbital angular momentum $\vec{L}_1$ with quantum number $l_1 = 2$, and a second electron has orbital angular momentum $\vec{L}_2$ with quantum number $l_2 = 3$. What are the possible quantum numbers for the total orbital angular momentum $\vec{L} = \vec{L}_1 + \vec{L}_2$? (3%)

7. Which of the following transitions in sodium do not occur as electric dipole transitions? (Hint: Give the selection rule that is violated.) (10%)

$$4S_{1/2} \rightarrow 3S_{1/2} \quad 4S_{1/2} \rightarrow 3P_{3/2} \quad 4P_{3/2} \rightarrow 3S_{1/2}$$
$$4D_{5/2} \rightarrow 3P_{1/2} \quad 4D_{3/2} \rightarrow 3P_{1/2} \quad 4D_{3/2} \rightarrow 3S_{1/2}$$

8. The probability distribution function for the speeds of the molecules in a classical ideal gas can be expressed by Maxwell distribution of molecular speeds:

$$n(v)dv = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/2kT} dv$$

where N and m are the number and mass of the molecules, respectively, k and T are the Boltzmann constant and temperature. Find the most probable speed. (10%)

9. Classically, the kinetic energy of rotation is

$$E = \frac{1}{2} I\omega^2$$

where I is the moment of inertia, ω the angular velocity of rotation.

(a) Find the energy levels of a diatomic molecule rotating about an axis through its center of mass. (5%)

(b) The energy difference ΔE between the $l = 0$ and $l = 1$ rotational levels in the CO molecule is found experimentally from measurement of the wavelength $\lambda = 2.6$ nm of the corresponding transition. For CO, ΔE is equal to 4.77×10^{-4} eV. Find the equilibrium separation, or bond length r_0, of the CO molecule. (5%) [Hint: the unified mass unit is $1\ u = 1.6605 \times 10^{-27}$ kg $= 931.4940 \times 10^6$ eV/c^2.]

10. Find (a) the current density, and (b) the drift velocity if there is a current 1 mA in a NO. 14 copper wire. [The diameter of NO. 14 wire is 0.064 in = 0.163 cm, and the electron density is 8.47×10^{28}/m^3] (5%)