

國立成功大學

112學年度碩士班招生考試試題

編 號：42

系 所：光電科學與工程學系

科 目：工程數學

日 期：0207

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Determine whether the given set of vectors is linearly dependent or linearly independent.

$$\mathbf{u}_1 = \langle 1, -2, 3, 4 \rangle, \mathbf{u}_2 = \langle 1, 4, 6, 8 \rangle, \mathbf{u}_3 = \langle 0, 1, 0, 0 \rangle, \mathbf{u}_4 = \langle 2, 5, 6, 8 \rangle$$

2. (10%) Find an orthogonal matrix \mathbf{P} that diagonalizes \mathbf{A} and the diagonal matrix \mathbf{D} such that $\mathbf{D} = \mathbf{P}^T \mathbf{A} \mathbf{P}$.

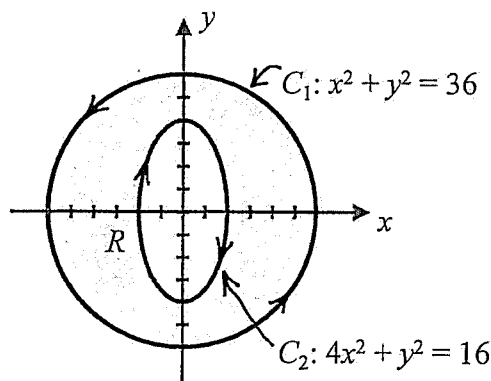
$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. (10%) Find values of a , b , and c so that the given matrix is orthogonal.

$$\begin{pmatrix} a & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{5}} \\ b & 0 & \frac{1}{\sqrt{5}} \\ c & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{5}} \end{pmatrix}$$

4. (10%) Find points on the surface $x^2 + 6x + y^2 + z^2 - 2z = 15$ at which the tangent plane is horizontal.

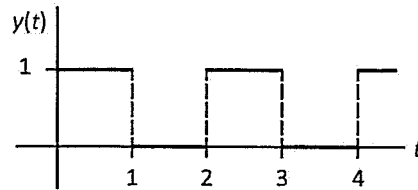
5. (10%) Evaluate $\oint_C (\cos x^2 - y) dx + \sqrt{y^2 + 1} dy$, where $C = C_1 \cup C_2$ is the shaded region R shown in the figure.



6. (20%) An object of mass 1 kg stretches a spring 10 cm. At $t = 0$ the mass is released from a point 100 cm below the equilibrium position with an upward velocity of 10 m/s. Determine the equation of free motion in the form of $x(t) = A \sin(\omega t + \varphi)$. Assume $g = 10 \text{ m/s}^2$. The differential equation for free undamped motion is $d^2x/dt^2 + (k/m)x = 0$.

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7. (10%) Find the Laplace transform of the periodic function shown in the following figure.



8. (20%) Solve Laplace's equation for the given boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$u(0, y) = 10y, \quad \left. \frac{\partial u}{\partial x} \right|_{x=1} = -1$$

$$u(x, 0) = 0, \quad u(x, 1) = 0$$