國立成功大學

113學年度碩士班招生考試試題

編　號：42

系　所：光電科學與工程學系

科　目：工程數學

日　期：0202

節　次：第 3 節

備　註：不可使用計算機

編號： 42

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please solve the following equation by Laplace transform.

$\frac{d^2y}{dt^2} + 4y = f(t),\ f(t) = u(t) - u(t-1)$

with initial condition: $y(0) = y'(0) = 0$

(a) Please solve $\hat{y}(s)$ (5 pts)

(b) Please solve $y(t)$ (10 pts)

2. Please solve the following equation by Laplace transform.

$y'' + 2y' + y = 0$

with initial condition: $y(0) = 1;\quad y'(0) = 0$

(a) Please solve $\hat{y}(s)$ (5 pts)

(b) Please solve y(t) (10 pts)

3. Please solve $\frac{\partial U}{\partial t} = \alpha^2 \cdot \frac{\partial^2 U}{\partial x^2}$, 0<x< l, 0<t<∞，boundary condition:

$\begin{cases} U(0,t) = A \\ U(l,t) = B \end{cases}$ 0<t<∞，A and B are constants, initial condition：

U(x,0)=ϕ(x), 0<x< l. (hint: the steady state solution is linear function) (20 pts)

4. (a) Evaluate the surface integral $\iint_S \vec{F} \cdot \vec{n}\, dA$ by the divergence theorem: $\vec{F} = [z - y,\ y^3,\ 2z^3]$, S the surface of $y^2 + z^2 \le 4,\ -3 \le x \le 3$. (10 pts)

(b) Calculate the line integral $\oint_C \vec{F} \cdot \vec{r}'\, ds$ by Stokes's theorem, clockwise as seen by a person sanding at the origin, for the following F and C. And assume the Cartesian coordinate to be right-handed.

$\vec{F} = [\cos \pi y,\ \sin \pi x,\ 0]$, around the rectangle with vertices (0, 1, 0), (0, 0, 1), (1, 0, 1), (1,1, 0).

(10 pts)

5. Please evaluate the following integral:

$\oint_C \frac{e^{2z}}{(z+1)^4}\, dz$, where C is $|z| = 3$ and counterclockwise. (10 pts)

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6. (a) Please find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{bmatrix}$$ (5 pts)

(b) Please find an eigenbasis (a basis of eigenvectors) and diagonalize the following matrix:

$$\begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$$ (5 pts)

7. (a) Please find all roots of $\sqrt[4]{-1}$ in the complex plane. (5 pts)

(b) Using Cauchy-Riemann equations, is the following function f is analytic?

$f(z) = u(x,y) + iv(x,y) = e^x(\cos y + i \sin y)$. (5 pts)