

## 成功大學 2005 光電所碩士班入學考--工程數學

1.

- (a) Give a definition for each of the following terms.
- Orthogonal matrix. (3%)
  - Unitary matrix. (3%)
  - Hermitian matrix. (3%)
  - Normal matrix. (3%)
  - Sturm-Liouville type of second order ordinary differential equation. (3%)
- (b) Proof that the property of Hermitian is preserved under unitary transform of a square matrix. (5%)
- (c) Proof that the trace of a square matrix is invariant under unitary transform. (5%)

2. Matrix  $A = \begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix}$ .

(a) Express the matrix function  $\frac{1}{1-xA}$  in a  $2 \times 2$  matrix form:  $\begin{pmatrix} f(x) & g(x) \\ g(x) & h(x) \end{pmatrix}$ .

(12%)

- (b) Describe the difference in the solving procedure and in result if replacing the matrix by  $\begin{pmatrix} 3 & -4 \\ 4 & -3 \end{pmatrix}$ . (3%)

3. Solve the first order ordinary differential equation:  $\frac{dy}{dx} = \frac{3y^2 + 2e^x}{2y - e^{3x}}$  with initial condition  $y(0) = 1$ . (15%) (hint: change variable to  $t = e^{-x}$ .)

4. Solve the following second order non-homogeneous differential equation:

$$\frac{d^2y}{dx^2} - 4y = 8e^{-2x} \text{ for } x > 0 \text{ with the initial conditions } \begin{cases} y(0) = 1 \\ y'(0) = 0 \end{cases}. \quad (15\%)$$

5. Solve the Laplace equation  $\nabla^2 \Psi(\rho, \phi, z) = 0$  in cylindrical polar coordinate for  $a < \rho < b$  with boundary conditions at two cylinders:  $\rho = a, b$   $\begin{cases} \Psi(a, \phi, z) = 0 \\ \Psi(b, \phi, z) = 1 \end{cases}$ . (15%)

6. Using residue theory calculate the definite integral  $\int_0^{\infty} \frac{1}{\sqrt[3]{x}(x+1)^2} dx$ . (15%)