1. (10 %) Figure 1 shows, in cross section, two solid spheres with uniformly distributed charge throughout their volumes. Each has radius R. Point P lies on a line connecting the centers of the spheres, at radial distance R/2.0 from the center of sphere 1. If the net electric field at P is zero, what is the ratio q_2/q_1 of the total charge q_2 in sphere 2 to the total charge q_1 in sphere 1?
2. (15 %) Consider a very long, hollow, metallic tube of rectangular cross section of dimension $a \times b$, as indicated in Figure 2. Three sides of the tube are kept at zero potential. The fourth side, insulated from the other three, is held at the variable potential $V = V_0 \sin(\pi x / a)$, where V_0 is a constant. Find the potential everywhere within the tube.
3. (15 %) A conducting bar slides at a constant velocity u along conducting rails in a region of uniform magnetic induction, as in Figure 3. The resistance in the circuit is R and the inductance is negligible.
   (a) Calculate the current I flowing in the circuit.
   (b) How much power is required to move the bar?
   (c) How does this power compare with the power loss in the resistance R?
4. (15 %) For a solenoid wound with N turns per unit length and carrying a current I, show that the magnetic induction on the axis is given by

   $B_z = \frac{\mu_0 NI}{2}(\cos\theta_1 + \cos\theta_2)$, where the angles are defined in Figure 4.
5. (15 %) A plane elliptically polarized wave results from the superposition of two plane-polarized waves whose $\boldsymbol{E}$ vectors are oriented in perpendicular directions and out of phase. For example, $\boldsymbol{E}$ can have the two components:

   $E_x = E_{x0} \exp j(\omega t - k z)$, $E_y = E_{y0} \exp j(\omega t - k z + \varphi)$, where E_{x0} and E_{y0} are real. Please show that the $\boldsymbol{E}$ and $\boldsymbol{H}$ vectors for an elliptically polarized wave are orthogonal if $\varphi = 0$ or if the wave impedance $\boldsymbol{E}/\boldsymbol{H}$ is real.
6. (5 %) Please explain the meaning of the Brewster Angle.
   (10 %) If light propagating in glass (with index of refraction 1.6) and incident on a glass-air interface, please calculate the critical angle of the total reflection and the Brewster angle.
7. (15 %) Ionized gases with equal electron and ion densities are called plasmas. If, in the plasma, the motion of the ions and the collision between the electrons and the gas atoms and molecules are ignored, please prove that the equivalent permittivity of the plasma is $\varepsilon = \varepsilon_0(1 - \frac{\omega_p^2}{\omega^2})$ (F/m), where $\omega_p = \sqrt{\frac{N e^2}{m \varepsilon_0}}$ (rad/s)

(背面仍有題目，請繼續作答)

is the plasma frequency, N is the density of the electrons, and ω is the wave frequency (rad/s).

(Hint: You can regard the plasma as a free electron gas and assume a time-harmonic electric field $\boldsymbol{E}$ in the x-direction at an angular frequency ω, and then write down the force equation for an electron with charge –e and mass m.)

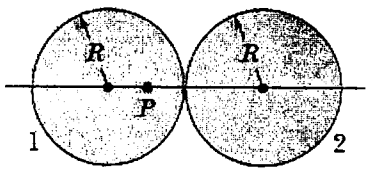


Fig. 1

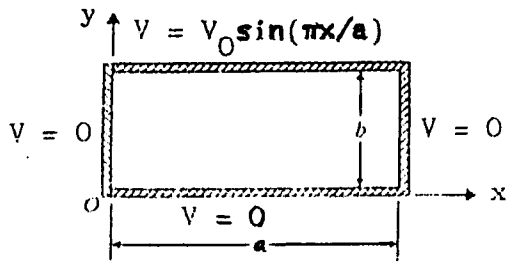
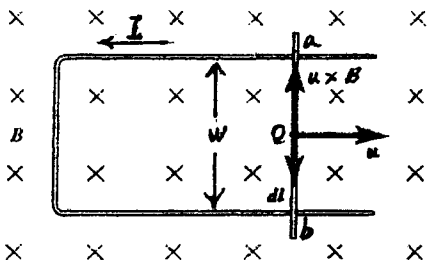


Fig. 2



A conducting wire **ab** slides with a velocity **u** along conducting rails in a region of uniform magnetic induction **B**. The magnetic force on the electrons in the wire produces a current **I** in the circuit. The electronic charge is taken to be **Q**.

Fig. 3

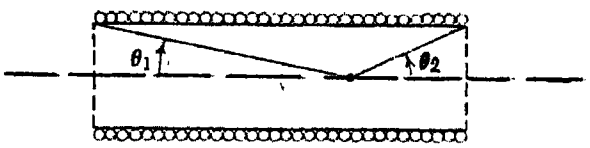


Fig. 4