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1. A given function $\chi''(v) = \dfrac{A}{1+(v-v_0)^2\tau^2}$, where A and τ are constants. Calculate the definite integral $\chi'(v) = \dfrac{1}{\pi} P \displaystyle\int_{-\infty}^{\infty} \dfrac{\chi''(v')}{v'-v} dv'$, where P stands for the Cauchy principal value of the integral that fallows.　(20%)

2. A set of coupling differential equation

$$\begin{cases} \dfrac{dy_1}{dx} = -i\dfrac{g}{2}y_2 \\ \dfrac{dy_2}{dx} = i\dfrac{g}{2}y_1 \end{cases}$$

where g is a constant. Solve this coupling differential equation with initial condition $y_1(0)$ and $y_2(0)$ at $x = 0$.　(20%)

3. Consider a classical nondriven oscillator whose equation of motion is given by

$$\dfrac{d^2y}{dt^2} + \kappa\dfrac{dy}{dt} + \omega_0^2 y = 0.$$

The variable y may correspond to the voltage across a parallel RLC circuit, in which case $\omega_0^2 = (LC)^{-1}$ and $\kappa = (RC)^{-1}$ are constants. Solve this differential equation with initial condition $y(t) = y(0)$ at $t = 0$.　(15%)

4. Set $AD - BC = 1$ and $A + D = 2\cos\theta$ in a matrix $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$. Express the matrix function M^S, where S is a positive integer. (Hint: taking advantage of A, B, C, D, and θ to express your answer.)　(15%)

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5. Evaluate the integrals along the path C that is the counterclockwise circle with $|z| = 4$.

(a) $\oint_C \dfrac{\sinh(3z)}{(z^2 + 4)} dz$　　(5%)

(b) $\oint_C \dfrac{e^z}{z^2(z - 2z - 3)} dz$　(5%)

6. Evaluate the Fourier Transform of the following functions:

(a) $f(x) = \exp(-\pi a^2 x^2)$　(5%)

(b) $f(x) = rect(x) = \begin{cases} 1 & \text{when } |x| \le \dfrac{1}{2} \\ 0 & \text{when } |x| > \dfrac{1}{2} \end{cases}$　(5%)

(c) $f(x) = rect\left|\dfrac{x - a}{a}\right| + rect\left|\dfrac{x + a}{a}\right|$,

where a is a constant (Hint: Use Convolution Theorem) (10%)