編號：H 52　系所：光電科學與工程研究所　科目：電磁學

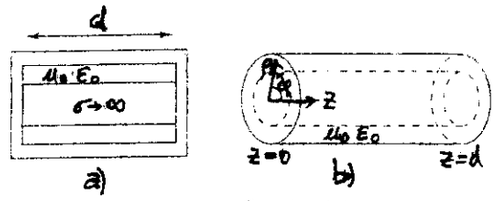
本試題是否可以使用計算機：□可使用，☑不可使用　（請命題老師勾選）

1. A static charge distribution produces a radial electric field $\vec{E} = A\frac{e^{-br}}{r^2}\vec{e}_r$, where A and b are constants.
   (a) (5 %) What is the charge density? Sketch it.
   (b) (5 %) What is the total charge Q?
   [Hint: You may use the property of the Dirac's delta function $\delta(\vec{r})$: $\nabla^2(1/r) = -4\pi\delta(\vec{r})$]

2. A point charge Q is at a distance d from the center of a grounded conducting sphere of a radius a ($a < d$). Determine
   (a) (10 %) the charge distribution induced on the surface of the sphere, and
   (b) (5 %) the total charge induced on the sphere.

3. Two coaxial circular turns of wire of radii a and b are separated by a distance x and carry currents i_a and i_b respectively. Assume $a >> b$.
   (a) (5 %) What is the mutual inductance?
   (b) (5 %) What is the force between the currents?

4. (15 %) Consider a resonator made of a section of coaxial line with length d and short circuited with conducting plates at both ends as shown below.



Coaxial resonator.

   What are the electric and magnetic fields for the TEM modes? Hint: The electric and magnetic fields for the TEM wave in an infinitely long coax, propagating along the positive z-direction take the form

$$E_\rho = \frac{E_0}{\rho}\cos(\kappa z - \omega t) \qquad H_\phi = \frac{E_0}{\eta\rho}\cos(\kappa z - \omega t)$$

5. (15 %) A plane polarized electromagnetic wave traveling in a dielectric medium of refractive index n is reflected at normal incidence from the surface of a conductor. Find the phase change undergone by its electric vector if the refractive index of the conductor is $n_2 = n(1 + i\rho)$.

（背面仍有題目，請繼續作答）

6. (10 %) A ferromagnetic sphere of radius a is magnetized with a magnetization $\vec{M}=(Az^2+B)\vec{e}_z$. Please determine the equivalent magnetization current densities $\vec{J}_m$ and $\vec{J}_{ms}$ and the equivalent magnetization charge densities ρ_m and ρ_{ms} (the center of the sphere is at the origin of the coordinate system).

7. An electron is introduced in a region of uniform electric and magnetic fields at right angles to each other ($\vec{E}=E\vec{e}_x, \vec{B}=B\vec{e}_y$).
   (a) (5 %) For what initial velocity will the electron move with constant velocity (both the direction and the magnitude of velocity are constant)?
   (b) (10 %) Consider a beam of electrons of arbitrary velocity distribution simultaneously injected into a plane normal to the electric field. Is there a time at which all the electrons are in this plane again?

8. (10 %) The mutual capacitance of two thin metallic wires lying in the plane $z = 0$ is C. Now the half space $z < 0$ is filled with a dielectric material with dielectric constant ε. What is the new capacitance?