編號： 51　系所：光電科學與工程研究所　　科目：近代物理

本試題是否可以使用計算機： ☑可使用 ， ☐不可使用 （請命題老師勾選）

Physical constants:

Speed of light in vacuum c = 2.998 x 10^{8} m/sec

Electron charge magnitude e = 1.602 x 10^{-19} coul

Planck's constant h = 6.626 x 10^{-34} joule-sec

Boltzmann's constant k = 1.381 x 10^{-23} joule/K

Avogadro's number N_0 = 6.023 x 10^{23} /mole

Coulomb's law constant $1/4\pi\varepsilon_0$ = 8.988 x 10^{9} nt-m^2/$coul^2$

Electron rest mass m_e = 9.109 x 10^{-31} kg = 0.511 MeV/c^2

Proton rest mass m_p = 1.672 x 10^{-27} kg = 938.3 MeV/c^2

Neutron rest mass m_n = 1.675 x 10^{-27} kg = 939.6 MeV/c^2

Atomic mass unit u = 1.661 x 10^{-27} kg = 931.5 MeV/c^2

Fine structure constant α = 7.30 x 10^{-3}

1 eV = 1.602 x 10^{-19} joule

1. Explain the following terms briefly (30 points, 5 points each)
   (a) Blackbody radiation
   (b) Wave-particle duality
   (c) Semiconductors
   (d) Meissner effects
   (e) Photomultiplier tube
   (f) Population inversion

2. The relativistic relationship between the total energy E of an electron, its momentum p and its rest mass m_e is $E^2 = p^2c^2 + m_e^2c^4$, where c is the speed of light in a vacuum. A photon of frequency ν is scattered, after colliding with an electron initially at rest, through an angle of 90°. Show that its frequency ν' after being scattered is given by $\nu' = \dfrac{m_e c^2}{h\nu + m_e c^2}\nu$. (15 points)

3. Electrons of energy 10 keV strike a tungsten target. The resulting X-rays are allowed to fall on a crystal which has a simple cubic structure with cube side equal to 5x10^{-10} m. It is found that, as the crystal is rotated, no X-rays are reflected unless the angle between the incident and reflected rays is greater than a certain value θ_0. Calculate θ_0. (15 points)

（背面仍有題目，請繼續作答）

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4. A particle of mass m moves in a one-dimensional harmonic potential, $V(x)=\frac{1}{2}kx^2$. (a) Show that $\psi(x)=A\exp(-\alpha x^2)$ is a solution of the appropriate Schrödinger equation provided that α has a particular value. (b) Deduce the corresponding value of the energy E of the particle, expressing it in terms of the frequency of oscillation ν of a classical particle of mass m moving under the influence of a spring of spring-constant k. (10 points)

5. A metal of volume V contains N electrons. The number of quantum states in an energy interval E to E + dE is given by $N(E)dE=\frac{8\pi V\left(2m^3\right)^{1/2}}{h^3}E^{1/2}dE$, where m is the electron mass and h is the Planck's constant. (a) Show that the Fermi energy is $E_F=\frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$. (b) The density of copper is 8.94 g/cm^3, and its atomic weight is 63.5. Assume that each copper atom contributes one free electron to the electron gas. Calculate the Fermi energy of copper. (10 points)

6. Given the electron of a hydrogen atom is in the state with n = 2 and l = 1. (a) Calculate the energy (without *L-S* coupling), the magnitudes of orbital and spin angular momentum. (b) Calculate all possible j and m_j. Draw the corresponding vector model diagrams. (10 points)

7. Consider an atom which is an example of *L-S* coupling, and an $nd\ n'd$ configuration of that atom in which both the ordering of the energy levels according to s', l', j', and the relative strengths of the s' and l' dependence of the energy, are normal. Draw a schematic energy level diagram for this configuration, using the same scale for the fine structure splitting of all the levels within a given multiplet. Label each level with the spectroscopic notation. (10 points)