編號： 53　系所：光電科學與工程研究所　　科目：電磁學

本試題是否可以使用計算機：□可使用 ，☑不可使用 （請命題老師勾選）

※單選題共 8 題佔 40 分、計算題共 6 題共 60 分，請於答案卷上依序列出答案※

1. 單選題： (總分: 40%, 每題: 5%)

(1) A thin, non-conducting ring of radius R, as shown in Fig. 1, which has a charge Q uniformly spread out on it. Now a small particle of mass m and charge –q is placed at point P and released. If R>>x, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to

(a) $\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}$ (b) $\sqrt{\frac{qQx}{4\pi\varepsilon_0 mR^4}}$

(c) $\frac{qQ}{4\pi\varepsilon_0 mR^3}$ (d) $\frac{qQx}{4\pi\varepsilon_0 mR^4}$

(e) $\sqrt{\frac{qQx}{4\pi\varepsilon_0 m}}\cdot\frac{1}{R^2+x^2}$

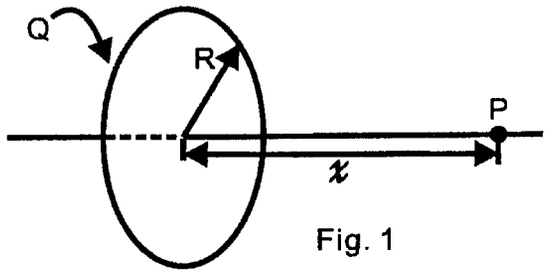
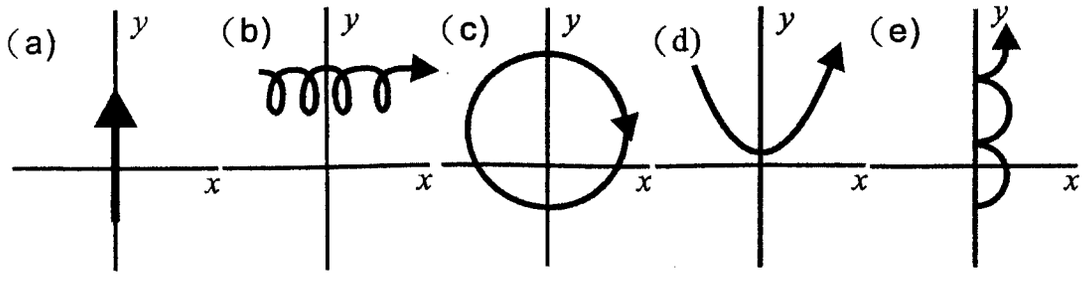


Fig. 1

(2) If $E_2 = E_1$, the tip of the electric field vector will describe a trajectory that, as viewed along the z-axis from positive z and looking toward the origin, is a

(a) line at 45° to the +x-axis
(b) line at 135° to the +x-axis
(c) clockwise circle
(d) counterclockwise circle
(e) random path

(3) A positively charged particle is moving in the xy-plane in a region where there is a non-zero uniform magnetic field B in the +z-direction and a non-zero, uniform electric field E in the +y-direction. Which of the following is a possible trajectory for the particle?



(4) The quantity B^2/μ_0 has units of:

(a) J (b) J/H (c) J/m (d) J/m^3 (e) H/m^3

(背面仍有題目,請繼續作答)

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(5) Two long conductors are arranged as shown above to form overlapping cylinders, each of radius r, whose centers are separated by a distance d, as shown in Fig. 2. Current of density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What are the magnitude and direction of the magnetic field at point A?

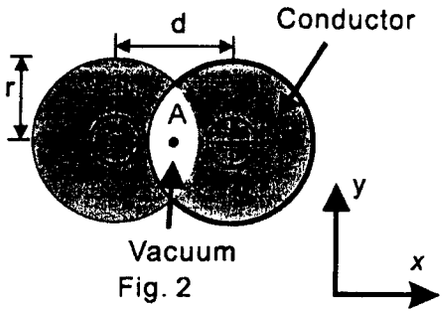


Fig. 2

(a) $(\mu_0/2\pi)\pi \cdot dJ$ in the +y-direction

(b) $(\mu_0/2\pi)d^2J/r$ in the +y-direction

(c) $(\mu_0/2\pi)4d^2J/r$ in the -y-direction

(d) $(\mu_0/2\pi)Jr^2/d$ in the -y-direction

(e) There is no magnetic field at A

(6) Two identical conducting spheres, A and B, carry equal charge. They are initially separated by a distance much larger than their diameters, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B is equal to

(a) 0 (b) F/16 (c) F/4 (d) 3F/8 (e) F/2

(7) Consider a novel material which has special properties such as $\mu < 0, \varepsilon < 0$. If an incident light coming from the bottom and enter the interface, which of the following will be the possible optical path shown in Fig. 3?

(a) (b) (c) (d)

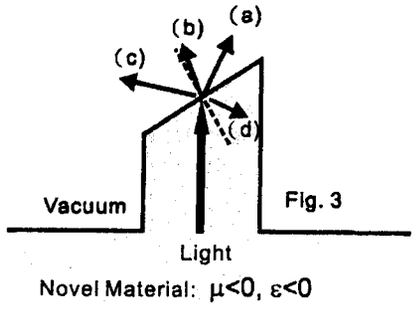


Fig. 3

(8) A copper penny slides on a horizontal frictionless table. There is a square region of constant uniform magnetic field perpendicular to the table, as shown in Fig. 4. Which graph correctly shows the speed υ of the penny as a function of time t?

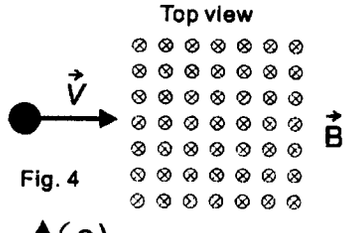
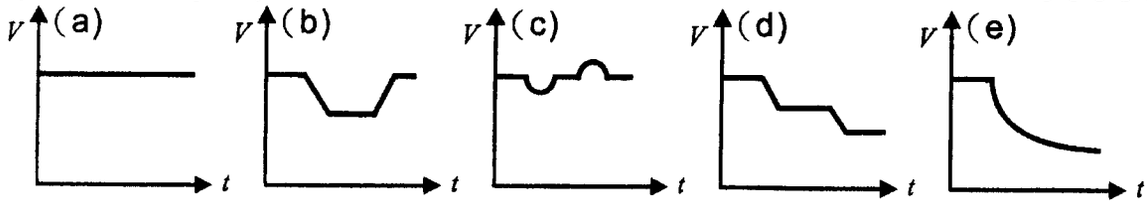


Fig. 4



2. (10%) As in Fig. 5, two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (susceptibility χ_e, mass density ρ). The inner one is maintained at a potential V, and the outer one is grounded. To what height (h) does the oil rise in the space between the tubes?

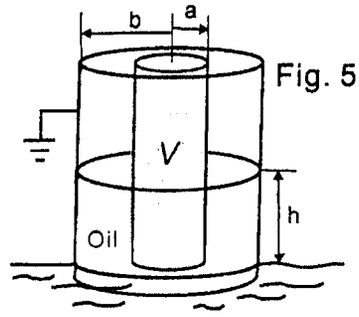


Fig. 5

3. (10%) A current I flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field **B** pointing out of the page, as shown in Fig. 6.

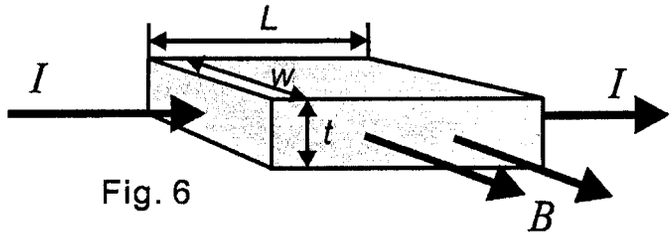


Fig. 6

(a) If the moving charges are positive, in which direction are they deflected by the magnetic field. This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electrical force to counteract the magnetic one. Equilibrium occurs when the two exactly canceled.

(b) Find the resulting potential difference between the top and bottom of the bar, in terms of B,υ (the speed of the charges), and the relevant dimensions of the bar.

4. (10%) The Maxwell's equation contains 4 different equations and can be expressed in both integral and differential form. Please write down the Maxwell's equation in both forms. Be sure to use MKS units, B for magnetic field, E for electric field, μ_0, and ε_0.

（背面仍有題目，請繼續作答）

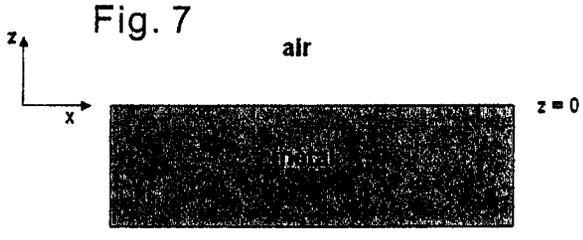
5. (10%) Consider a uniform plane wave propagating along the z-direction in free space with the electric field $\vec{\xi} = -\hat{x}\, E_{xm} \sin(\omega t - kz) + \hat{y}\, E_{ym} \cos(\omega t - kz)$. Please derive the Poynting vector, the time-average Poynting vector, the instantaneous field energy density, and the time-average field energy density.

6. (10%) Assume a surface wave exists at the metal-air interface as shown in figure 7 and the magnetic fields $\vec{H}$ in region I and II have the following forms:

Region I: $\vec{H} = A\, e^{i(kx-\omega t)}\, e^{-\alpha_0 z}\, \hat{e}_y$

Region II: $\vec{H} = B\, e^{i(kx-\omega t)}\, e^{\alpha_m z}\, \hat{e}_y$.

Fig. 7



The dielectric constants in air and metal are ε_0 and $\varepsilon(\omega)$, respectively.

(a) According to Maxwell's equations, please write down the electric field $\vec{E}$ in region I and II.

(b) Please write down the boundary conditions of $\vec{E}$ and $\vec{H}$ at the metal-air interface.

(c) According to (b), please write down the dispersion relation of this surface wave.

7. (10%) Suppose the magnetic field on the axis of a right circular cylinder is given by $\vec{B} = B_0\left(1 + \nu \cdot z^2\right)\hat{e}_z$. Suppose the θ-component of $\vec{B}$ is zero inside the cylinder.

(c) Calculate the radial component of the field $B_r(r,z)$ for points near the axis.

(d) What current density $\vec{J}(r,z)$ is required inside the cylinder if the field described above is valid for all radii r?