1. A real square matrix is shown as $A=\left[a_{jk}\right]$, which transpose matrix and inverse matrix are A^T and A^{-1}, respectively.

(a) Please answer what relations must be satisfied among A, A^T and A^{-1} when matrix A is symmetric, skew-symmetric, or orthogonal, respectively. (5%)

(b) If matrix A is shown as $A=\begin{bmatrix} m & 0 \\ 0 & n \end{bmatrix}$, please find e^{At}? (10%)

2.

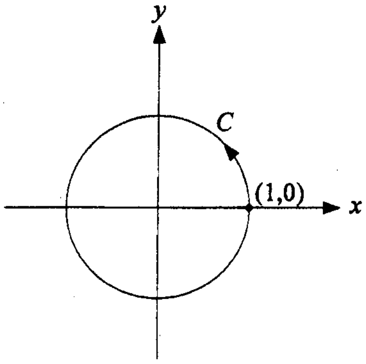
(a) Find the Fourier series representation of

$$f(x)=\begin{cases} 0, & -\pi < x \le 0 \\ x, & 0 \le x < \pi \end{cases} \qquad (10\%)$$

(b) From the Fourier expansion show that

$$\frac{\pi^2}{8}=1+\frac{1}{3^2}+\frac{1}{5^2}+\cdots. \qquad (10\%)$$

3. Please apply Green's theorem to evaluate $\oint_C (3xdy-5ydx)$, the contour C is a circle and shown below. (15%)



4. The differential equation $m\frac{d^2x}{dt^2}+b\frac{dx}{dt}+kx=0$ can be used to describe a damped simple harmonic motion. Its solution can be written as the form of $x(t)=x_m e^{-\alpha t}\cos(wt+\phi)$, where x_m is the amplitude of the damped oscillator. Please solve this differential equation and find the α and w in terms of m, b, k (20%).

5. The binomial distribution is $P(m)=\dfrac{n!}{m!(n-m)!}p^m(1-p)^{n-m}$. In the limit $n\to\infty$, $p\to 0$, and $np=a$, find the new distribution $P(m)$ (Hint: use $\lim\limits_{n\to\infty}\left(1-\dfrac{a}{n}\right)^n=e^{-a}$) (10%)

6. Using theorem of residues, calculate $\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\frac{e^{i\omega t}}{\omega_0^2-\omega^2+i\alpha\omega}$ $(\alpha>0)$ for

(a) t<0 (b) t>0 (20%)