Physical constants:

Avogadro's number: $N_a=6.02\times10^{23}$ particles/mol

Coulomb constant: $k=8.987\times10^{9}\ N\cdot m^2/C^2$

Mass of electron: $m_e=9.1\times10^{-31}$ Kg

Mass of unit: $u=1.66\times10^{-27}$ Kg

Speed of light: c=299792458 m/s

Fine structure constant: $\alpha=7.297\times10^{-3}$

Permeability of free space: $\mu_0=4\pi\times10^{-7}\ N/A^2$

Boltzmann's constant: $k=1.38\times10^{-23}$ J/K

Fundamental charge: $e=1.6\times10^{-19}$ C

Mass of proton: $M_p=1.67\times10^{-27}$ Kg

Planck's constant: $h=6.6\times10^{-34}\ J\cdot s$

Constant of gravitation: $G=6.67\times10^{-11}N\cdot m^2/Kg^2$

Gas constant: $R=8.3\ J/mol\cdot K$

1. (10%) Provide a brief qualitative description for each item listed below:
   (a) Heisenberg uncertainty principle
   (b) The difference between bosons and fermions

2. (10%) An X-ray photon of wavelength 6pm that collides with an electron is scattered by an angle 90^{o}. (a) What is the change in wavelength of the photon? (b) What is the kinetic energy of the scattered electron?

3. (15%) A particle is confined to a two-dimensional box defined by the following boundary conditions: $V(x,y) = 0$, for $-L/2 \le x \le L/2$ and $-3L/2 \le y \le 3L/2$; and $V(x,y) = \infty$ elsewhere. (a) Determine the energies of the lowest three bound states. (b) Identify the quantum numbers of the lowest doubly degenerate bound state and determine its energy.

4. (6%) During the photoelectric effect experiment, sodium metal is illuminated with light of wavelength 4.20×10^{2} nm. The stopping potential is found to be 0.65 V. When the wavelength is change to be 3.10×10^{2} nm, the stopping potential is found to be 1.69 V. The speed of the light, $c=3.00\times10^{8}$ m/s, and the elementary charge, $e=1.60\times10^{-19}$ C. Find a value for Plank's constant.

5. (9%) An electron is captured in a potential of the form: $V = \infty$ for $x \le 0$ and for $x \ge a$, $V = 0$ for $0 < x < a/2$ and $V = V_0$ for $a/2 < x < a$. Draw the potential, the wave function for the ground state where $E < V_0$, and the second excited state where $E > V_0$.

(背面仍有題目,請繼續作答)

6. (10%) Explain the following nouns as clear as possible

(a) Pauli exclusion principle.

(b) Zeeman effect.

7. A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose it is in an energy eigenstate $\psi(x) = (\gamma^2/\pi)^{1/4}\exp(-\gamma^2x^2/2)$ with energy $E = \hbar^2\gamma^2/2m$.

(a) (3%) Find the mean position of the particle.

(b) (4%) Find the mean momentum of the particle.

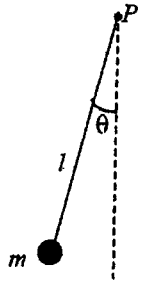
(c) (5%) Find $V(x)$.

(d) (8%) Find the probability $P(p)dp$ that the particle's momentum is between p and $p+dp$.

8. A mass m is attached by a massless rod of length l to a pivot P and swings in a vertical plane under the influence of gravity (as shown in the figure).

(a) (5%) In the small angle approximation find the energy levels of the system.

(b) (5%) Find the lowest order correction to the ground state energy resulting from inaccuracy of the small angle approximation.



9. (a) (5%) What is the quantum mechanical Hamiltonian for a free electron with magnetic moment μ in the external constant magnetic field H_z in the z-direction, in the reference frame of the electron?

(b) (5%) Suppose that an extra constant magnetic field H_y is imposed in the y-direction. Determine the form of the quantum mechanical operator for the time rate of change of μ in this case.