※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. (10%) A wire is bent into the shape of the quarter circle C given by

$$x = 2\cos(t),\ y = 2\sin(t),\ z = 3 \text{ for } 0 \le t \le \pi/2.$$

The density function of the wire is $\delta(x,y,z) = xy^2$ grams/centimeter. Find the center of mass of the wire.

2. (10%) (a) Find the eigenvalues of the matrix $\begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$, (b) corresponding to the eigenvalues obtained in (a), find the eigenvectors.

3. (10%) Use the Laplace transform to solve the differential equation: $ty'' + (4t - 2)y' - 4y = 0$; $y(0) = 1$.

4. (5%) (a) Find the first five nonzero terms of the power series solution of the initial value problem, about the point where the initial conditions are given: $y' + e^x y = x^2$; $y(0) = 4$.

(7%) (b) Consider a surface Σ of an elliptical cone given by its coordinate function $x = au\cos(v)$, $y = bu\sin(v)$, and $z = u$. Find the equation of the tangent plane to the Σ at point $P_0\left(\frac{a\sqrt{3}}{4}, \frac{b}{4}, \frac{1}{2}\right)$.

5. (8%) Suppose a damped force harmonic motion, under the influence of a periodic driving force $f(t) = A\cos(\omega t)$, with A and ω positive constants, is governed by the following spring equation

$$y'' + \frac{c}{m}y' + \frac{k}{m}y = \frac{A}{m}\cos(\omega t).$$

Find a complete particular solution of this equation in terms of $\omega_0 = \sqrt{k/m}$, ω, A, c and m.

6. (15%) Find the Fourier Series of the function $f(x) = |x|$, $-\pi < x < \pi$, $f(x + 2\pi) = f(x)$

7. (15%) Find $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}$ by using Residue theorem. First show the poles in the complex plane, and then do the complex integral.

8. (20%) Use the method of Fourier transform to solve $\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial u}{\partial t}$, $-\infty < x < \infty$, $0 \le t < \infty$, with $u(x,0) = \delta(x - x_0)$ and $u(x,t) \to 0$ as $x \to \pm\infty$