系所組別 光電科學與工程研究所甲組
考試科目 近代物理

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Physical constants:

Speed of light in vacuum c = 2.998 x 10^8 m/sec
Electron charge magnitude e = 1.602 x 10^{-19} coul
Planck's constant h = 6.626 x 10^{-34} joule-sec
Boltzmann's constant k = 1.381 x 10^{-23} joule/K
Avogadro's number N_0 = 6.023 x 10^{23} /mole
Coulomb's law constant $1/4\pi\varepsilon_0$ = 8.988 x 10^9 nt-m^2/$coul^2$
Electron rest mass m_e = 9.109 x 10^{-31} kg = 0.511 MeV/c^2
Proton rest mass m_p = 1.672 x 10^{-27} kg = 938.3 MeV/c^2
Neutron rest mass m_n = 1.675 x 10^{-27} kg = 939.6 MeV/c^2
Atomic mass unit u = 1.661 x 10^{-27} kg = 931.5 MeV/c^2
Fine structure constant α = 7.30 x 10^{-3}
1 eV = 1.602 x 10^{-19} joule

1. The average threshold of scotopic vision is 4.00×10^{-11} W/m^2 at a central wavelength of 500 nm. (a) If light having this intensity and wavelength enters the eye and the pupil is open to its maximum diameter of 8.50 mm, how many photons per second enter the eye? (5%) (b) What is the temperature of a black body that would radiate most intensely at this central wavelength? (5%)
2. A bullet (m = 0.01 kg) has a velocity of 400 m/s, accurate to within 0.01%. (a) Calculate the de Broglie wavelength for the bullet.(5%) (b) Within what limits could we determine the position of the bullet along the direction of the velocity? (5%)
3. (a) What can the phenomena of photoelectric effect be not explained by wave theory? (5%) (b) Lithium, beryllium, and mercury have work functions of 2.30 eV, 3.90 eV, and 4.50 eV, respectively. A blue light is incident on each of these metals. Determine which metals exhibit the photoelectric effect. (5%)
4. Based on Rutherdord's model, the distance of closest approach of the particle to the center of the nucleus, R, is

$$R = \frac{D}{2}\left[1 + \frac{1}{\sin(\phi/2)}\right],$$

Where D is a convenient parameter qeual to the distance of the closest approach to the nucleus in a head-on collision and ϕ is the scattering angle. Give physical arguments explaining the results with $\phi \to \pi$ and $\phi \to 0$.(10%)

(背面仍有題目,請繼續作答)

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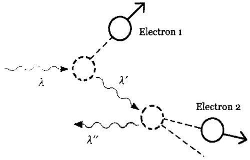
5. In Compton's experiment the wavelength of the incident photon is λ. The photon occurs twice scattering process, producing a photon having wavelength λ'' and moving in a direction directly opposite the original photon. Determine the numerical value of $\Delta\lambda = \lambda'' - \lambda$. (10%)



6. Show that for a free particle of mass m moving in one dimension, the function $\psi(x) = A\sin(kx) + B\cos(kx)$ is a solution to the time-independent Schrödinger equation for any values of the constant A and B. (10%)
7. If the angular momentum of Earth in its motion around the Sun were quantized like a hydrogen electron according to $L = mvr = n\hbar$, what would Earth's quantum number be? (5%) How much energy would be released in a transition to the next lowest level? (4%) Would that energy release (presumably as a gravity wave) be detectable? (3%) What would be the radius of that orbit? (3%) (The radius of Earth's orbit is 1.5×10^{11} m.)
8. The relative binding of the extra electron in the arsenic atom that replaces an atom in silicon or germanium can be understood from a calculation of the first Bohr orbit of this electron in these materials. Four of arsenic's outer electrons from covalent bonds, so the fifth electron sees a singularity charged center of attraction. This model is a modified hydrogen atom. In the Bohr model of the hydrogen atom, the electron moves free space at a radius a_0 given by

$$a_0 = \frac{\epsilon_0 h^2}{\pi m_e e^2}$$

When an electron moves in a crystal, we can approximate the effect of the other atoms by replacing ϵ_0 with $\kappa\epsilon_0$ and m_e with an effective mass for the electron. For silicon κ is 12 and the effective mass is about $0.2m_e$, and for germanium κ is 16 and the effective mass is about $0.1m_e$.

(a) Estimate the Bohr radii for the outer electron as it orbits the impurity arsenic atom in silicon and germanium. (5%)

(b) Find the ionization energies of the doping arsenic atoms for silicon and germanium. (5%)

9. A particle of mass m is in a infinite square well potential given by

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$$V(x) = \begin{cases} 0, \text{ for} & -\frac{a}{2} < x < \frac{a}{2} \\ \infty, \text{ for } x < \frac{-a}{2}, & x > \frac{a}{2} \end{cases}$$

Since this potential is symmetric about the origin, the probability density $|\psi(x)|^2$ must also be symmetry.

(a) Show that this implies that either $\psi(-x) = \psi(x)$ or $\psi(-x) = -\psi(x)$. (5%)

(b) Show that the proper solutions of the time-independent Schrödinger equation can be written (5%)

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{a}} \cos\left(\frac{n\pi x}{a}\right) \text{ for } n = 1, 3, 5, \ldots, & -a/2 \le x \le a/2 \\ \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \text{ for } n = 2, 4, 6, \ldots, & -a/2 \le x \le a/2 \\ 0 \quad \text{for} \quad x \le -a/2, & x \ge a/2 \end{cases}$$

(c) Find the allowed energies in the infinite square well. (5%)