※ 考生請注意：本試題 ☑可 ☐不可 使用計算機

1. Consider a rectangular waveguide, infinitely long in the x-direction, with a width (y-direction) 2 cm and a height (z-direction) 1 cm. The walls are perfect conductor (Fig. 1).

(a) What are the boundary conditions on the components of $\vec{B}$ and $\vec{E}$ at the walls? (5 %)

(b) Write the wave equation which describes the $\vec{B}$ and $\vec{E}$ fields of the lowest mode. (5 %)

(c) For the lowest mode that can propagate, find the phase velocity and the group velocity. (5 %)

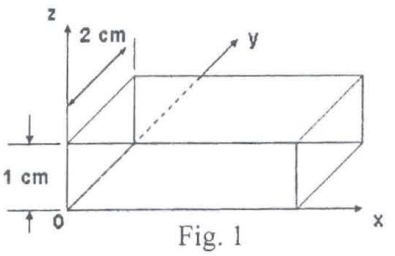

Fig. 1

2. Consider a TM plane wave incident obliquely from an isotropic medium with permittivity ε and permeability μ upon another isotropic medium with permittivity ε_t and permeability μ_t (See Fig. 2).

(a) Please derive the reflection and transmission coefficients. (5 %)

(b) Please derive the conditions of total reflection. Please derive the field components of $\vec{B}$ and $\vec{E}$ at region t when total reflection occurs. (10 %)

(c) If both ε_t and μ_t are negative values, please derive and plot the directions of wave vector, Poynting vector and electric field vector of the transmitted wave. (5 %)

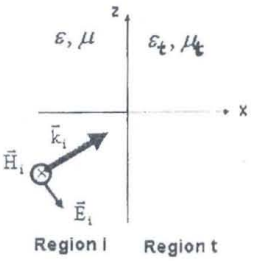

Fig. 2

3. (a) Consider a wave packet that consists of two traveling waves having equal amplitude and slightly different angular frequencies $\omega_0 + \Delta\omega$ and $\omega_0 - \Delta\omega$ ($\Delta\omega << \omega_0$). Please derive the group velocity u_G of this wave packet. (5 %)

(b) Consider plasma with the free electron charge q_e, mass m_e and density N_e. Assume the plasma is collisionless (no collision loss). Please derive the effective permittivity of the plasma in terms of plasma frequency $\omega_p = \sqrt{N_e q_e^2 / (m_e \varepsilon_0)}$. (5 %)

(c) Prove the following relation between group velocity u_G and phase velocity u_P in the plasma of part (b): $u_G = u_P - \lambda (d u_P / d\lambda)$, where λ is the wavelength. (5 %)

(背面仍有題目,請繼續作答)

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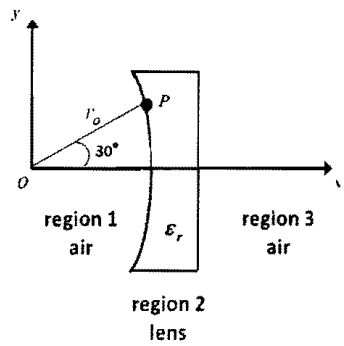
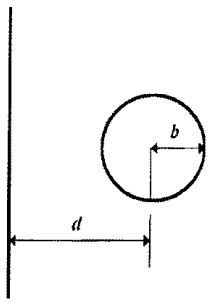
4. Dielectric lenses can be used to collimate EM fields. In the plot below, the left surface of the lens is that of a circular cylinder, and the right surface is a plane. If $\vec{E}_1$ at point $P\,(r_0, 30°, z)$ in region 1 is $5\bar{a}_r - 3\bar{a}_\phi$, what must be the dielectric constant of the lens in order that $\vec{E}_3$ in region 3 is parallel to the x-axis. (15%)



5. An uncharged sphere of radius b is placed in an initially uniform electric field $\vec{E}_0 = \bar{a}_z E_0$ in air. Determine the electric potential $V(R,\theta)$ and the electric field intensity $\vec{E}(R,\theta)$ both inside and outside the sphere after its introduction if (20%)
   (a) the sphere is made of conductor, assuming $V(b,0) = 0$. (10%)
   (b) the sphere is made of dielectric with a dielectric constant ε_r. (10%)

6. A very long, straight wire and a conducting circular loop of radius of b are arranged as shown below. ($\int_0^{\pi} \frac{d\theta}{p + q\cos\theta} = \frac{\pi}{\sqrt{p^2 - q^2}}$) (15%)



   (a) Determine the mutual inductance between them. (5%)
   (b) Find the force on the circular loop that is exerted by the magnetic field due to an upward current I_1 in the long straight wire. The circular loop carries a current I_2 in the counterclockwise direction. (5%)
   (c) Assume that the circular loop is rotated about its horizontal axis by an angle α, find the torque exerted on the circular loop. (5%)