系所組別： 太空天文與電漿科學研究所
考試科目： 電磁學
考試日期：0219，節次：2

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1. Ampere's law in Maxwell's equations is

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad (1)$$

where $\vec{B}$ is the magnetic field, $\vec{J}$ is the current density and $\vec{E}$ is the electric field.

(a) Take the divergence of the Ampere's law. **(10%)**

The continuity equation for the charge density ρ is

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \vec{J} = 0 \qquad (2)$$

(b) Please use Eq.(2) to discuss the relation between the result of question (a) and the Poisson's equation $\nabla \bullet \vec{E} = 4\pi\rho$. **(10%)**

(c) Take divergence of the Faraday's law in Maxwell's equations and discuss its relation to the statement that there are no magnetic monopoles, i.e., $\nabla \bullet \vec{B} = 0$. **(10%)**

2. There is a one dimensional slab conductor that has a resistivity η. The thickness of the slab conductor is $2a$, (see Fig.1 below). The Ohm's law is $\vec{E} = \eta \vec{J}$.

(a) Derive an equation for the diffusion of the magnetic field in the conductor using Maxwell's equation. (You may neglect the displacement current in the Ampere's law.) **(10%)**

(b) A steady state spatially uniform magnetic field $\vec{B} = B_0\hat{z}$ is maintained in the region where $x \geq a$, and $\vec{B} = -B_0\hat{z}$ in the region where $x \leq -a$. Here, $\hat{z}$ is the unit vector in the z direction. What is the magnetic field profile, i.e., $\vec{B}(x)$ inside the conductor using the equation derived in (a)? **(10%)**

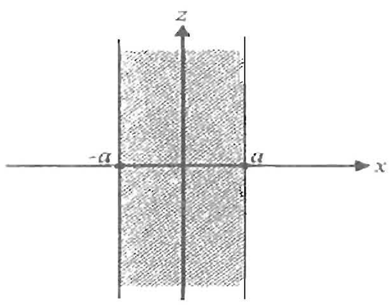

Fig. 1: A schematic diagram of a slab conductor

(背面仍有題目,請繼續作答)

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3. The figure 2 below is a sectional view of two circular coils with radius a, each wound with N turns of wire carrying a current I, circulating in the same direction in both coils. The coils are separated by a distance a equal to their radii. In this configuration the coils are called **Helmholtz coils**; they produce a very uniform magnetic field in the region between them.

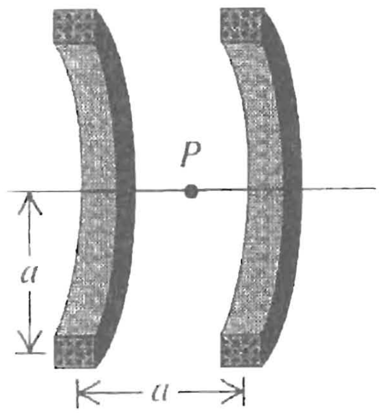

Fig. 2: Configuration of a Helmholtz coil

(a) Derive the expression for the magnitude B of the magnetic field of a point on the axis a distance x to the right of point P, which is midway between the coils. **(10%)**

(b) Calculate $\frac{dB}{dx}$ and $\frac{dB}{dx}$ at point **P** ($x = 0$). Discuss how your results show that B is very uniform in the vicinity of **P**. **(10%)**

4. Consider the region enclosed on three sides by the grounded conducting planes shown in figure 3. The end plate on the right has a potential $V = V_0 \sin(\frac{5\pi}{b} y)$. All planes are assumed to be infinite in extent in the z-direction. Find the potential distribution within the region. **(14%)**

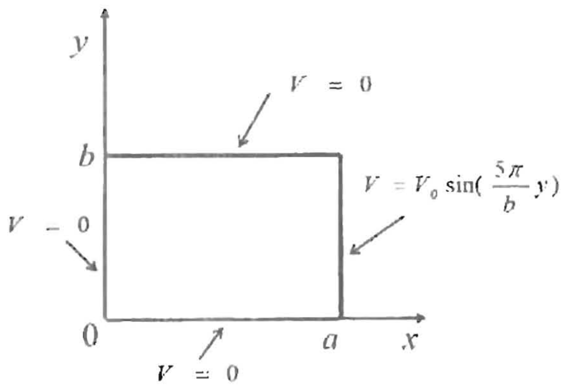

Fig. 3

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5. Airplanes and trains move through the earth's magnetic field at rather high speeds, so it is reasonable to wonder whether this field can have a reasonable effect on them. We shall use a typical value of 0.5 Gauss for the earth's magnetic field.

(a) The train of Taiwan High Speed Rail reaches speeds of up to 300 km/hour and moves on tracks roughly 1.5 m apart. At top speed moving perpendicular to the earth's magnetic field, what potential difference is induced across the tracks as the wheels roll? Does this seem large enough to produce noticeable effects? **(8%)**

(b) A Boeing 747-400 aircraft has a wingspan of 64.4 m and a cruising speed of 900 km/hour. If there is no wind blowing (so that this is also the plane's speed relative to the ground), what is the maximum potential difference that could be induced between the opposite tips of the wings? Does this seem large enough to cause problems with the plane? **(8%)**