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Calculation processes have to be described.

I. (i) Solve a differential equation $\frac{dx}{dy}+\frac{3x+y}{x-y}=0$. (6 percent)

(ii) Get a general solution of $\left(\frac{d^2}{dx^2}-2\frac{d}{dx}+2\right)y=\exp(2x)\cdot\sin(x)$ (12 percent)

(iii) Calculate Fourier transform of

$$f(x)=\begin{cases}1-x^2 & (|x|\le 1)\\ 0 & (|x|>1)\end{cases}$$ (10 percent)

II. (i) Calculate $\nabla(1/r)$ ($r\neq 0$). Here, $r\equiv\sqrt{x^2+y^2+z^2}$, $\nabla=\boldsymbol{i}\partial/\partial x+\boldsymbol{j}\partial/\partial y+\boldsymbol{k}\partial/\partial z$. $\boldsymbol{i},\boldsymbol{j}$ and $\boldsymbol{k}$ are basis vectors in directions of x, y and z, respectively. (4 percent)

(ii) Calculate $\nabla\cdot(\nabla(1/r))$ ($r\neq 0$ and $r=0$). Hint: Use Gauss theorem $\int_\Omega \mathrm{div}\,\boldsymbol{f}\,dV=\int_\Sigma \boldsymbol{f}\cdot\boldsymbol{n}\,dS$ and Dirac's delta function $\delta(\boldsymbol{r})$, $\int\delta(\boldsymbol{r}-\boldsymbol{r}_0)f(\boldsymbol{r})dV=f(\boldsymbol{r}_0)$. (8 percent)

(iii) Evaluate a Green function G of Poisson equation $\nabla'^2G(\boldsymbol{r},\boldsymbol{r}')=-\delta(\boldsymbol{r}-\boldsymbol{r}')$. (6 percent)

III. Calculate an inverse matrix of $A=\begin{pmatrix}-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}}\\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{pmatrix}$. (8 percent)

(背面仍有題目,請繼續作答)

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IV. Calculate the following definite integaral by employing residue theorem.

$\int_0^{\infty} \frac{\sin x}{x} dx$ (16 percent)

V. Solve a two-dimensional Laplace equation under the boundary condition given below.

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}\right)u=0 \quad (0<x<a, 0<y<b),$$

Boundary condition

$$\left.\frac{\partial u}{\partial x}\right|_{x=0}=0, \quad \left.\frac{\partial u}{\partial x}\right|_{x=a}=\sin\left(\frac{\pi}{b}y\right),$$

$$u|_{y=0}=0, \quad u|_{y=b}=0.$$

(16 percent)

VI. Find the eigenvalues and normalized three eigen vectors of the matrix shown below.

$$A=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$ (14 percent)