※ 考生請注意：本試題不可使用計算機

- Calculation processes have to be described.
- System of unit used in the problems of this examination is SI unit system unless stated.
- Characters representing physical constants listed up below are available if necessary: Elementary charge: e [C], permittivity of vacuum: ε_0 [m^{-3} kg^{-1} s^4 A^2](or [$F \cdot m^{-1}$]), permeability of vacuum: μ_0 [m kg s^{-2} A^{-2}] (or [$H \cdot m^{-1}$]), speed of light in vacuum: c [m s^{-1}].

I. Problems about electrostatic field (25 %)

i. Electric charge fills a space in the region of $0 < x < d$ uniformly (electric charge density is ρ). Find the spatial profile of the electric field $E(x)$. The electric field E for $x < 0$ is zero. Here, x is a coordinate of a Cartesian coordinate system. (5 %)

ii. There are concentric two spheres, whose radii are a and b ($< a$). Electric charge +Q (Q > 0) is uniformly distributed and fixed on the surface of the sphere with a radius of a, and electric charge −Q is uniformly distributed and fixed on the surface of the sphere with a radius of b. Find the spatial profile of the scalar potential ϕ as a function of radius r. Use the following three conditions:

1). $\phi \longrightarrow 0$ for $r \longrightarrow \infty$. 2). ϕ is finite as $r \longrightarrow 0$. 3). ϕ is continuous at $r = a$ and $r = b$.

(8 %)

iii. Three positive point charges q_A, q_B and q_C are put and fixed on the line l with the same intervals a. Evaluate the Coulomb force acting to each point charge when an enough thin and sufficiently wide conducting plate is put at the middle point between B and C vertically to the line l as shown in Fig. 1. Positive direction of the force is defined as the direction from A to C.

(4% each, 12%)

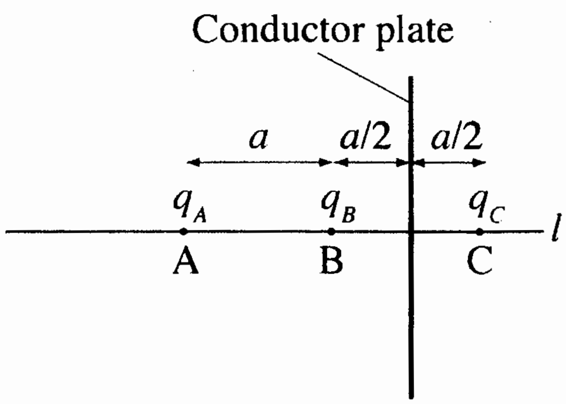

Fig. 1

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II. Electric field in dielectric medium: (5 % each, total 25 %)

As shown in Fig. 2, the region between two electrodes of a parallel-plate capacitor is filled with two dielectric media 1 and 2, whose thicknesses and dielectric constants are d_1, d_2 and ε_1, ε_2, respectively. The potential difference between the two electrodes is $\Delta\phi$. Find the electric fields (E_1, E_2) and electric flux densities (D_1, D_2) in each dielectric medium. Also, find the surface density of polarization charge at the interface between the dielectric media 1 and 2, σ_p.

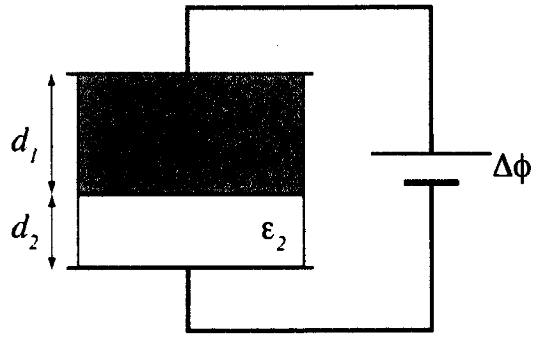

Fig. 2

III. Cyclotron radiation: (15 %)

Radiation power W (per unit solid angle dΩ) emitted from a charged particle having a uniform circular motion is expressed by the following equation. Electric charge of the particle is q and the rotation center and the rotation plane are the origin and the x-y plane, respectively as shown in Fig. 3.

$$\frac{dW}{d\Omega} = \frac{q^2}{16\pi^2\varepsilon_0 c^3}\left|\frac{\boldsymbol{R}}{R}\times\left(\frac{\boldsymbol{R}}{R}\times\frac{d\boldsymbol{v}}{dt}\right)\right|^2 .$$

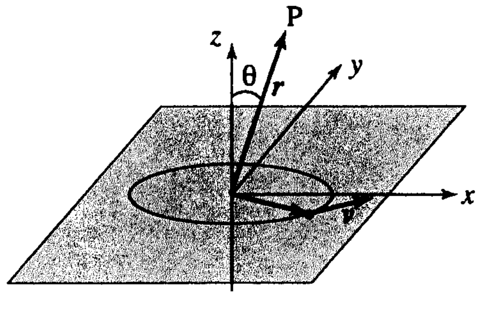

Fig. 3

Here, $\boldsymbol{R}$ and $\boldsymbol{v}$ are and the position vector of the observation point P and the velocity vector of the charged particle, respectively. Rewrite dW/dΩ as a function of ω, θ, and a.

Here, ω, θ and a are the angular frequency of the circular motion, the zenith angle of the observation point and the radius of the circular motion, respectively. [Hint: $\boldsymbol{R}/R$ can be written as $\boldsymbol{R}/R = (\sin\theta, 0, \cos\theta)$ without loss of generality.]

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IV. Interface between different media: (10 % each, total 20 %)

Two kinds of dielectric medium (1 and 2) contact at an interface. A plane electromagnetic wave is injected perpendicularly into the interface plane. Find the reflection and transmission coefficients of the electromagnetic wave for the following situation. The reflection rate is defined as a ratio between energy fluxes the reflected wave of and the incident wave. The transmission rate is defined as a ratio between energy fluxes of the transmitted wave and the incident wave:

Use the following parameters: electric permittivity ε_1 and ε_2, permeability $\mu_1=\mu_2=\mu_0$. The suffices 1 and 2 represent the two kind of the dielectric media.

V. A plane electromagnetic wave propagates in z direction in vacuum, whose x and y components of the wave electric field are written in the following forms, respectively: (15 %)

$E_x(z,t) = f_1(z-ct) + g_1(z+ct)$,

$E_y(z,t) = f_2(z-ct) + g_2(z+ct)$.

Here, $f_{1,2}(u)$, $g_{1,2}(u)$ are functions of only u. $c = 1/(\varepsilon_0\mu_0)^{1/2}$.

(i) Derive the x and y components of the wave magnetic field $B_x(z,t)$ and $B_y(z,t)$. (10 %)

(ii) Find the z component of the Poynting vector S_z. (5 %)