※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please answer the following questions:

   (a) $\dfrac{d}{dx} y^{\sqrt{x}} = ?$ (5pts)

   (b) $\dfrac{d}{dx} \displaystyle\int_{b(x)}^{0} dy\, F(x,y) = ?$ (5pts)

   (c) $\dfrac{d}{dx} \displaystyle\int_{0}^{a(x)} dy\, F(x,y) = ?$ (5pts)

   (d) What are the first three terms of the Taylor series of $\sin x$ at $x = 0$? (5pts)

2. (a) Solve the differential equation
$$\frac{dV}{dx} = \pm\, \frac{1}{2}\, \frac{\sin x}{\sqrt{k^2 - \cos x}},$$
   where $k^2 \geq 0$ is a parameter. (10pts)

   (b) Determine the integration constant by imposing that $V = 0$ when $k^2 = \cos x$ for $k^2 \leq 1$ and the condition that V must be continuous across the $k^2 = 1$ boundary for $k^2 > 1$. (5pts)

   (c) Give a rough sketch of V as a function of x when k^2 varies from 0, to 1 to a number larger than 1. (5pts)

3. There is a two-dimensional vector $\vec{T} = a(x,y)\,\hat{x} + b(x,y)\,\hat{y}$ in three-dimensional (x,y,z) Cartesian coordinates, where the unit vectors in x, y, z directions are $\hat{x}$, $\hat{y}$, and $\hat{z}$ respectively, please calculate

   (a) $\vec{\nabla} T$ (5pts)

   (b) $\vec{\nabla} \times \vec{T}$ (5pts)

   (c) $\hat{z}\,\hat{z} \cdot \vec{\nabla} \times \vec{T}$, where $\hat{z}\,\hat{z}$ is a tensor and $\cdot$ denotes dot product. (5pts)

4. Perform the integral $\displaystyle\int_{-\infty}^{\infty} dx\, \frac{e^{itx}}{x^2 + 1}$ for a real value t. (15pts)

5. (a) Please show that $f(x + ct)$ and $f(x - ct)$ are solutions to the equation
$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}. \qquad\qquad (1)$$
   (10pts)

   (b) What is the difference between these two solutions? (5pts)

   (c) Show that $g = e^{i(kx - \omega t)}$ is also a solution to Eq. (1) if frequency ω is a function of wave vector k. Find that relation between ω and k. (10pts)

   (d) Using the relation obtained in (c), show that function g is a special case of function f, i.e., find the function f for the solution g. (5pts)