國立成功大學 104 學年度碩士班招生考試試題

系所組別：太空與電漿科學研究所

考試科目：電磁學　　　　考試日期：0211，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

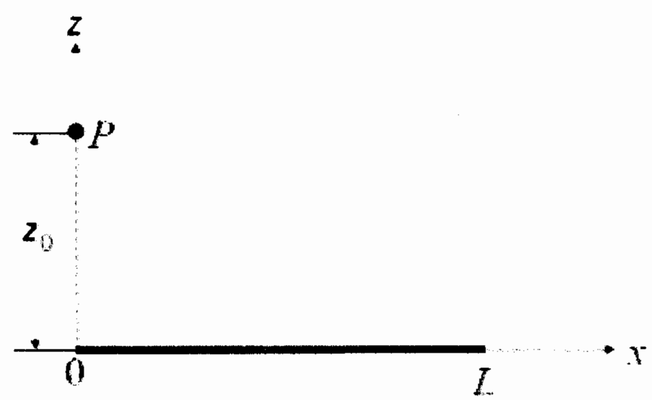
1. Electric charge is distributed non-uniformly along a thin line segment that lies in the x-axis between $x=0$ and $x=L$, as shown in the figure below. The charge per unit length λ is:

$$\lambda(x)=\begin{cases}\dfrac{\lambda_0 x}{L} & \text{for } 0\le x\le L\\ 0 & \text{otherwise}\end{cases}$$

   (a) Find the total charge in the line segment. (5%)

   (b) Find the vector electric field at Point P, which has the coordinates $(x,z)=(0,z_0)$. (14%)

   (c) Check you results in Part (b) and show that those results are consistent with what is expected for the case $z_0 \gg L$. (6%)



2. (a) Gauss's law in differential form is:

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}. \tag{2.1}$$

   Starting with Eq. (2.1), perform a step-by-step derivation of Gauss's law in integral form. (5%)

   (b) Under steady-state consideration, Ampère's law in differential form is:

$$\nabla\times\mathbf{B}=\mu_0\mathbf{J}. \tag{2.2}$$

   Starting with Eq. (2.2), perform a step-by-step derivation of Ampère's law in integral form. (5%)

   (c) An infinite, thin line charge with uniform charge per unit length λ lies on the z-axis of the cylindrical coordinates. Make use of the result in Part (a) to calculate the electric field vector at an arbitrary point (r,ϕ,z) for $r\neq 0$. (5%)

   (d) The infinite, thin line charge in Part (c) now moves with a vector $\mathbf{v}=v\hat{z}$. Make use of the result in Part (b) to calculate the magnetic field vector at an arbitrary point (r,ϕ,z) for $r\neq 0$. (5%)

3. Three charged particles A, B and C are between two fixed, parallel charged plates of infinite area. One of the plates has uniform positive charge and the other has uniform negative charge, resulting in a uniform electric field between them. The potential difference between the two plates is 10 volts. We let the electric potential $\phi = 0$ be at the negative plate, and $\phi = 10$ V at the positive plate. The three charged particles are momentarily at the midpoint between the two plates, as shown in the figure below. The information about these particles, including their kinetic energy (KE) when they are at the midpoint between the plates, is as follows:

   Particle A: electric charge $+e$ (e is the elementary charge), mass m_A, zero KE;
   Particle B: electric charge $+e$, mass m_B, zero KE;
   Particle C: electric charge $-e$, mass m_C, KE$=3\,$eV moving toward the negative plate.
   It is given that $m_A > m_B > m_C$.

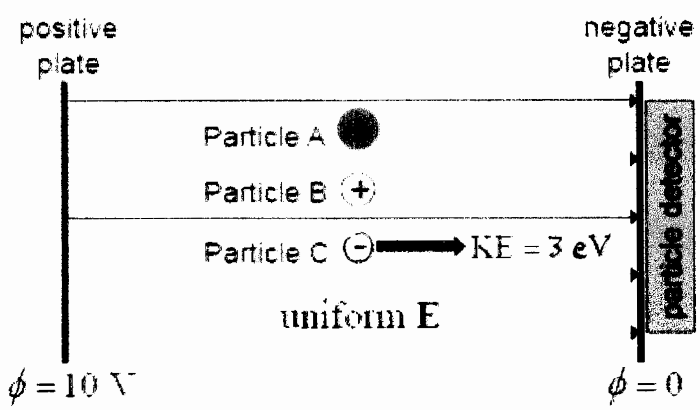
   (a) Give the total (kinetic + potential) energy of each charged particle. (5%)

   A particle detector is set up at the negative plate to observe all the charged particles that reach there.

   (b) Which of the three charged particles will reach the particle detector first? Which one will be the second to reach the particle detector? (2%)

   (c) Discuss what will happen to the third charged particle (the particle that is not in your answers in Part (b)) by describing its motion. (5%)

   (d) Explain the reason(s) for your answers in Part (b). (3%)



4. Given two inertial frames, Frame S and Frame S', where Frame S' moves with a constant velocity $\mathbf{v}$ with respect to Frame S, the rules of Lorentz transformation for electric and magnetic fields between the two frames are as follows:

$$E_{\parallel}' = E_{\parallel} \; ; \quad \frac{\mathbf{E}_{\perp}'}{c} = \gamma\left(\frac{\mathbf{E}_{\perp}}{c} + \boldsymbol{\beta}\times\mathbf{B}_{\perp}\right) \; ;$$

$$B_{\parallel}' = B_{\parallel} \; ; \quad \mathbf{B}_{\perp}' = \gamma\left(\mathbf{B}_{\perp} - \boldsymbol{\beta}\times\frac{\mathbf{E}_{\perp}}{c}\right) ,$$

where the unprimed quantities are fields in Frame S, the primed quantities are fields in Frame S', the $\parallel$ and $\perp$ subscripts respectively refer to directions parallel and perpendicular to $\mathbf{v}$, the vector $\boldsymbol{\beta} = \mathbf{v}/c$, c is the speed of light, and $\gamma = \left(1-\beta^2\right)^{-1/2}$.

A particle of charge q and velocity $\mathbf{u} = u\hat{z}$ is momentarily at the origin. Make use of the rules of Lorentz transformation to find the electric field and magnetic field at locations (x, y, z) everywhere except for the origin at that instant of time. Assume the particle to be a point charge. Also, assume that $u^2 \ll c^2$ in your calculation and neglect terms of the order of $(u/c)^2$ and smaller. (20%)

5. Consider a large region with a uniform magnetic field $\mathbf{B} = B\hat{z}$ but no electric field. The equation of motion for a particle of electric charge q and mass m is:

$$m\frac{d\mathbf{v}}{dt} = q\mathbf{v}\times\mathbf{B} \quad . \tag{5.1}$$

(a) Solve Eq. (5.1) to obtain the velocity $\mathbf{v} = (v_x, v_y, v_z)$ and position $\mathbf{x} = (x, y, z)$ of the particle as a function of time t, using the following initial conditions at $t = 0$: $\mathbf{x} = (0,0,0)$ and $\mathbf{v} = (v_{\perp}, 0, 0)$. (17%)

(b) Give an expression for the cyclotron frequency Ω of the particle by identifying the frequency of the particle's periodic motion. (3%)