國立成功大學104學年度碩士班招生考試試題

系所組別：太空與電漿科學研究所

考試科目：應用數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

- **Answers must be described logically and straightforwardly so that readers can follow easily.**
- **Calculation processes have to be described.**

I. Answer the following questions (30 percent):

(i) Find solutions of $x^3 - 1 = 0$. (6 percent)

(ii) Find a general solution of the following differential equation by the use of Laplace transform:

$$\frac{d^2}{dt^2} f(t) + 3\frac{d}{dt} f(t) + 2f(t) = \exp(-3t), \quad f(0) = \left.\frac{d}{dt} f(t)\right|_{t=0} = 0,$$ (8 percent)

(iii) Calculate the Fourier transform of the following functions and draw a graph ($F(\omega)$ vs ω) of each result. (8 points each, total 16 points)

(a) $f(t) = \begin{cases} 1 & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$

(b) $f(t) = \exp\left(-\frac{t^2}{2}\right)$.

II. Sturm–Liouville boundary value problem: Calculate the eigen values λ and eigen functions of the following differential equation with the boundary conditions (BCs): (Hint: Consider the three cases $\lambda > 0$, $\lambda = 0$, $\lambda < 0$, separately.)

$$\frac{d^2y}{dx^2} + \lambda y = 0, \text{ B.C. } y(0) = \left.\frac{dy}{dx}\right|_{x=\pi} = 0.$$ (16 percent)

III. Calculate the following definite integral by employing residue theorem.

$$\int_{-\infty}^{\infty} \frac{1}{x^6 + a^6} dx, \qquad a > 0.$$ (16 percent)

IV. (i) Find $\nabla(1/r)$ $(r\neq0)$. Here, $r=\sqrt{x^2+y^2+z^2}$, $\nabla=\boldsymbol{i}\partial/\partial x+\boldsymbol{j}\partial/\partial y+\boldsymbol{k}\partial/\partial z$. $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are basis vectors in directions of x, y and z, respectively. (6 percent)

(ii) Calculate $\nabla\cdot(\nabla(1/r))$ $(r\neq0$ and $r=0)$. Hint: Use the Gauss theorem $\int_\Omega \mathrm{div}\,\boldsymbol{f}\,dV=\int_\Sigma \boldsymbol{f}\cdot\boldsymbol{n}\,dS$ and Dirac's delta function $\delta(\boldsymbol{r})$, $\int\delta(\boldsymbol{r}-\boldsymbol{r}_0)f(\boldsymbol{r})dV=f(\boldsymbol{r}_0)$.

(6 percent)

(iii) Prove that the Green function of the Helmholtz equation $(\nabla\cdot\nabla+k^2)\psi(\boldsymbol{r})=0$, ($k$: real) is given as $G(\boldsymbol{r},\boldsymbol{r}')=-\frac{1}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}\exp(ik|\boldsymbol{r}-\boldsymbol{r}'|)$, by substituting this Green function into the Helmholtz equation.

(10 percent)

V. Find the eigenvalues and <u>normalized</u> three eigen vectors of the matrix shown below (2 percent each, total 12 percent). From those, find a matrix B, that can diagonalize the matrix T by a similarity transformation (4 percent).

$$T=\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}.$$

(16 percent)