

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Use Maxwell equations to calculate the propagation speed of an electromagnetic wave in vacuum. (10 points)
2. Positive electric charges distribute on a perfect ellipsoid conductor. The ellipsoid can be described by the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ in (x,y,z) Cartesian coordinates with non-zero parameters a , b , and c . What are the directions of the electric field at the points $(a,0,0)$, $(0,b,0)$, and $(0,0,c)$ on the ellipsoid? (10 points)
3. The magnetic flux through a thin circular conducting wire is changing with time. The rate of change is $\dot{\Phi}$. The radius of the circle formed by the wire is d . What is the electric field inside the wire? (10 points)
4. The magnetic field \mathbf{B} can be cast in terms of scalar potentials α and β in such a way: $\mathbf{B} = \nabla\alpha \times \nabla\beta$. (a) Show that $\mathbf{A} = \alpha\nabla\beta$ and $\mathbf{A}' = -\beta\nabla\alpha$ are two vector potentials for \mathbf{B} . (10 points) (b) Show that the difference of these two potentials can be expressed as $\nabla\chi$, and find the expression for χ . (10 points)
5. A charged particle moves in uniform static magnetic \mathbf{B} , and electric \mathbf{E} fields experiencing the Lorentz force $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}/c)$, where q is the charge of the particle, \mathbf{v} is the particle velocity, and c is the speed of light. It is further assumed that \mathbf{E} is perpendicular to \mathbf{B} . (a) Please write down the equations of motion for the charge particle using $\mathbf{F} = m\mathbf{a}$ in the direction parallel to the magnetic field, and in the directions perpendicular to the magnetic field in Cartesian coordinates, where m is the mass of the particle, and \mathbf{a} is the acceleration. The equations in the directions perpendicular to the magnetic field are coupled. You are free to choose the directions of \mathbf{B} , and \mathbf{E} . (10 points) (b) Please solve the equation of motion first with $\mathbf{E} = 0$ case. You are free to choose the convenient initial conditions. (20 points) (c) Find the particular solution to the equation of motion when $\mathbf{E} \neq 0$ utilizing the facts that both \mathbf{B} and \mathbf{E} are spatially uniform, and are not functions of time. (20 points)