國立成功大學 105 學年度碩士班招生考試試題

系　　所：太空與電漿科學研究所

考試科目：應用數學

考試日期：0227，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Show your calculation steps carefully. A correct answer, unsupported by calculations, explanation, or algebraic work will not receive credit; partial credit will be given for correct steps.

1. **(10%)** Consider the following matrix :

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\\ 5 & 6\end{bmatrix}$$

Find the eigenvalues of A^TA and also of AA^T. For both matrices find a complete set of orthonormal eigenvectors.

2. **(10%)** Evaluate the following sum:

$$\sum_{k=2}^{n}\frac{1}{k^2-1}$$

3. **(20%)** Find the general solution to the following differential equations:

(a) (10%) $(2x+y)y'=\frac{4x^2}{y}+y+4x$

(b) (10%) $y''+5y'+6y=12e^t+6$

4. **(10%)** Evaluate the following integral:

$$\int_0^\infty \frac{x^3}{e^x-1}dx$$

[Hint: $\xi(4)=\sum_{n=1}^{\infty}\frac{1}{n^4}=\frac{\pi^4}{90}$]

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5. **(15%)** Let $f(t)$ be a periodic signal of period 1. One says that $f(t)$ has a ***half-wave symmetry*** if

$$f(t-\frac{1}{2}) = -f(t)$$

(a) (5%) Sketch an example of a signal that has ***half-wave symmetry***.

(b) (10%) If $f(t)$ has half-wave symmetry and its Fourier series is

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{i\,2\pi nt}$$

Show that $C_n = 0$ if n is even.

6. **(10%)** Prove that $\nabla \cdot (\nabla\psi \times \nabla\varphi) = 0$, where ψ and φ are scalar-valued function of (x, y, z).

7. **(10%)** Calculate $\iint_R (x+y)^2 e^{x-y} dxdy$ where *R* is the region bounded by $x+y=1$, $x+y=4$, $x-y=-1$, and $x-y=1$.

8. **(15%)** Consider the integral operator $K f(x) \equiv \int_0^1 (x^2 + y^2) f(y) dy$

(a) (10%) Find all eigenvelues of this operator.

(b) (5%) Find all eigenfunctions corresponding to non-zero eigenvalues