國立成功大學106學年度碩士班招生考試試題

系　所：太空與電漿科學研究所

考試科目：應用數學

考試日期：0213，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

[1] (A total of 20 points)

Obtain solutions $X(T)$ and $V(T)$ for the following simultaneous ordinary differential equations. For **(a)**, **(b)**, and **(c)**, use common initial conditions

$$X(0) = 0 \quad \text{and} \quad V(0) = 1$$

In **(b)**, $a > 0$ is a real number.

(a) (4 points)

$$\frac{dV}{dT} = -X$$

and

$$\frac{dX}{dT} = V$$

(b) (8 points)

$$\frac{dV}{dT} = -X - aV$$

and

$$\frac{dX}{dT} = V$$

(c) (8 points)

$$\frac{dV}{dT} = -X + \sin T$$

and

$$\frac{dX}{dT} = V$$

[2] (A total of 16 points)

We define Fourier transform of a function $f(x)$ by

$$\hat{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ikx} f(x) dx$$

Obtain Fourier transform for a Gaussian function

$$f(x) = \exp\left(-\frac{x^2}{2}\right)$$

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[3] (A total of 14 points)

(a) (7 points) Solve the following simultaneous equations for x, y, z, and w:

$$x - y + 2z + w = 9$$
$$2x + y - z + 3w = 6$$
$$x + 3y + 2z - 2w = 2$$
$$-3x + z + 4w = -3$$

(b) (7 points) Solve the following simultaneous equations for x, y, z, and w:

$$x + 4y + 2z + 3w = 1$$
$$2x + 3y + 4z + w = -2$$
$$3x + 2y + z + 4w = 3$$
$$4x + y + 3z + 2w = 0$$

[4] (A total of 14 points)

(a) (7 points) The imaginary unit is given by i. Prove Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

(b) (7 points) Obtain the following definite integral

$$\int_0^{\pi} \frac{d\theta}{1 - 2a\cos\theta + a^2} \qquad (a : \text{real number}, a \neq 0, \pm 1)$$

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[5] (A total of 16 points)

The unit vectors in the direction of the x, y, and z axes of a three dimensional Cartesian coordinate system are given by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

(a) (8 points) Consider a volume V in a domain $x \geq 0$, which is surrounded by three surfaces $z = 0$, $z = 1$, and $x^2 + y^2 = 1$. After drawing the the shape of the volume, obtain a volume integral

$$\int_V \nabla \cdot \mathbf{A} dV'$$

for a vector $\mathbf{A} = x^2\hat{\mathbf{i}} + 2xy\hat{\mathbf{j}} + 2yz\hat{\mathbf{k}}$.

(b) (8 points) Obtain a line integral

$$\oint \mathbf{A} \cdot \mathbf{dl}$$

for a vector $\mathbf{A} = \cos(x)\hat{\mathbf{i}} + x\left[2 - \sin(y)\right]\hat{\mathbf{j}}$ along a circle $x^2 + y^2 = 4$ on the xy-plane.

[6] (A total of 20 points)

(a) (10 points) Obtain a solution $U = U(t, x)$ for a wave equation

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}$$

in a domain $-\pi/2 \leq x \leq \pi/2$. Employ boundary conditions $U(t, -\pi/2) = 0$ and $U(t, \pi/2) = 0$, and an initial condition $U(0, x) = \cos(x)$. Draw x versus $U(0, x)$, $U(\pi/2, x)$, and $U(\pi, x)$ in a single figure.

(b) (10 points) Obtain a solution $U = U(t, x)$ for a diffusion equation

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2}$$

in a domain $-\pi/2 \leq x \leq \pi/2$. Employ boundary conditions $U(t, -\pi/2) = 0$ and $U(t, \pi/2) = 0$, and an initial condition $U(0, x) = \cos(x)$. Draw x versus $U(0, x)$, $U(1, x)$, and $U(\infty, x)$ in a single figure.