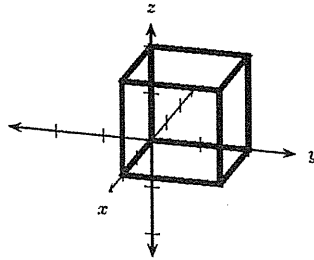※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1.

(a) Consider the function $f(r, \theta, \phi) = r^2 \cos\theta$. Determine $\vec{v} = \nabla f$ and show that $\nabla \times \vec{v} = 0$. **(10%)**

(b) Let $\vec{v} = xy^2 \hat{x}$, Show that this satisfies the divergence theorem over the following cube with sides of length 2. **(10%)**


2. The Electric Field Cage generates DC and low-frequency AC fields for sensor design, calibration, and evaluation. Consider the following design: a region enclosed on three sides by the grounded conducting planes shown in following figure. The end plate on the right has a potential $V = V_0$, All planes are assumed to be infinite in extent in the z-direction. Find the potential distribution $V(x,y)$ within the region **(10%)**, and calculate the electric field at the central point $P(a/2, b/2)$ **(10%)**.
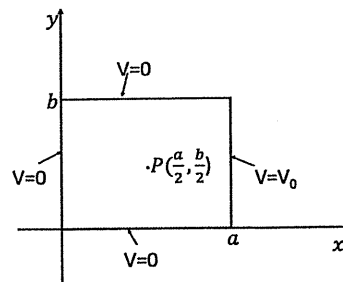
3. Two square capacitor plates in air have separation d, sides of length b, and charge $\pm Q$ as illustrated. Fringing fields can be neglected.

(a) What is the capacitance C_a of this device? **(4%)**
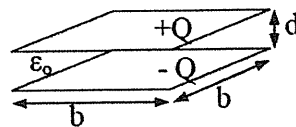
(b) A perfectly *conducting* plate is introduced d/10 between the capacitor plates, leaving parallel gaps of width d/10 above and below itself. What now is the device capacitance C_b when it is fully inserted? **(6%)**
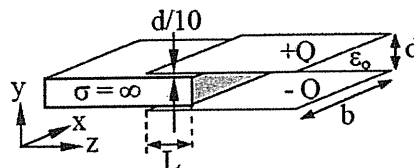

(c) What is the magnitude and direction of the force $\vec{f}$ on the new plate of Part (b) as a function of the insertion distance L. Please express your answer as a function of the parameters given in the figure. **(10%)**

4. A uniform line charge λ is placed on an infinite straight wire (parallel to the x axis), a distance H above a grounded conducting plane at $z = 0$ as shown:
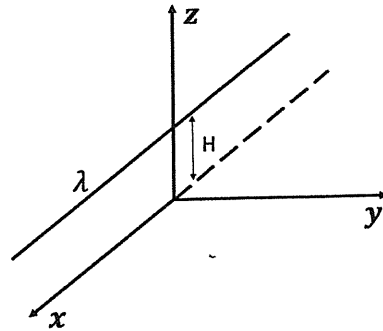
(a) What is $\nabla^2 V$ for $z > 0$ including on the location of the wire? **(7%)**

(b) The potential V can be calculated by adding an image potential contribution V_I to the line charge potential contribution V_R: $V = V_R + V_I$. What is V_I if we fix the convention $V_I = 0$ along the line defined by $\{z = 0, y = 0\}$? Express your final answer in terms of the shown Cartesian coordinates. **(13%)**

5.

(a) Suppose the inductance L is due to the self-inductance of a toroidal coil with rectangular cross section (inner radius a, outer radius b, and height h as shown in the figure below) carrying a total of N turns. Express L in terms of N, h, a, and b. **(10%)**
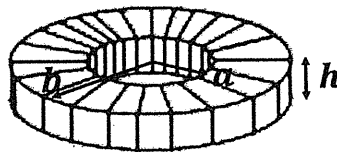
(b) Suppose this toroidal coil has a magnetic field $\vec{B}(s, \phi, z) = f(s)\hat{\phi}$, where s is a cylindrical coordinate variable: e.g. the inner radius is described as $s = a$. In this magnetic field, suppose a pointlike magnetic dipole with $\vec{m} = m\hat{z}$ is placed at rest at $s = u \in (a, b)$ inside the torus. What are the forces and torques on the dipole due to the magnetic field? **(10%)**