※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Please answer the following questions:

   (a) $\dfrac{d}{dx}y^{x^{1/3}} =?$ (5pts)

   (b) $\dfrac{d}{dx}\int_{x^2}^{0} dy\, G(x,y) =?$ (5pts)

   (c) $\dfrac{d}{dx}\int_{0}^{x^3} dy\, G(x,y)=?$ (5pts)

   (d) What are the first three terms of the Taylor series of $\cos x$ for $x < 1$? (5pts)

2. (a) Solve the differential equation
   $$\frac{dV}{dx} = \pm\, \frac{1}{2}\,\frac{\sin x}{\sqrt{k^2 - \sin^2(x/2)}},$$
   where $k^2 \geq 0$ is a parameter. (10pts)

   (b) Determine the integration constant by imposing that $V = 0$ when $k^2 = \sin^2(x/2)$ for $k^2 \leq 1$, and that V is continuous at $k^2=1$. (5pts)

   (c) Give a rough sketch of V as a function of x when k^2 varies from 0, to 1 to a number larger than 1. (5pts)

3. There is a two-dimensional vector $\vec{T}= (x-y)\,\hat{x}+ (x+y)\,\hat{y}$ in three-dimensional (x,y,z) Cartesian coordinates, where the unit vectors in x, y, z directions are $\hat{x}$, $\hat{y}$, and $\hat{z}$ respectively, please calculate

   (a) $\vec{\nabla}T$ (5pts)

   (b) $\vec{\nabla}\times\vec{T}$ (5pts)

   (c) $\hat{y}\,\hat{z}\,\bullet\,\vec{\nabla}\times\vec{T}$, where $\hat{y}\,\hat{z}$ is a tensor and $\bullet$ denotes dot product. (5pts)

4. Perform the integral $\displaystyle\int_{-\infty}^{\infty} dx\,\frac{1}{\left(x^2+1\right)^2}$ for real x . (15pts)

5. Please find the Fourier series of a function $f(x)= \sin^3 x + \cos^2 x$. (10pnts)

6. Please find all three cubic roots of the following numbers: -8 and 1+i. Here, $i = \sqrt{-1}$. (10pnts)

7. Please show that $g = e^{i(kx-\omega t)}$ is a solution to Eq.(1) if frequency ω is a function of the wave vector k. Find that relation. (10pnts)

$$\frac{\partial^2 f}{\partial t^2} = c^2\,\frac{\partial^2 f}{\partial x^2}. \qquad\qquad (1)$$