國立成功大學

111學年度碩士班招生考試試題

編　號：54

系　所：太空與電漿科學研究所

科　目：電磁學

日　期：0219

節　次：第 1 節

備　註：不可使用計算機

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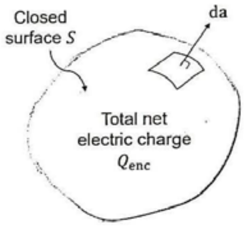
1. Consider an arbitrary closed surface S. Starting with one of the Maxwell's equations in MKS units, derive the integral form of Gauss's law:

$$\int_S \mathbf{E} \cdot \mathbf{da} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

where the integration is over the entire closed surface S, $\mathbf{da}$ is the vector area of an infinitesimal element of S in the direction outwardly normal to the surface, $\mathbf{E}$ is the electric field, Q_{enc} is the total net electric charge enclosed by the surface, and ϵ_0 is the permittivity of free space. (10%)



2. A sphere of radius R consists of three layers, as shown in the figure below. Each layer is made of non-conducting materials and is spherically symmetric about the center of the sphere. The inner layer, which is a sphere of radius $R/4$, is neutral in terms of electric charge. The middle layer, which corresponds to the range of radial distances from the center between $R/4$ and $R/2$, has a uniform charge density. The total electric charge in the middle layer is Q. The rest of the sphere outside the middle layer belongs to the outer layer, which also has a uniform charge density. The total electric charge in the outer layer is $-Q$.

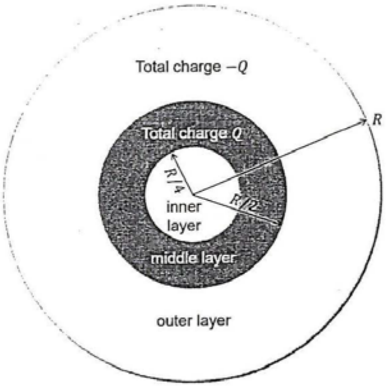
(a) Find the charge density of the middle layer, ρ_{mid}. (4%)

(b) Find the charge density of the outer layer, ρ_{out}. (4%)

In order to indicate locations, we use a spherical coordinate system with the origin at the center of the sphere and r corresponding to the radial distance from the origin.

(c) Find the electric field vector for all locations with $r > R$ (i.e. everywhere outside the sphere of radius R). (5%)

(d) Find the electric field vector for all locations with $r \leq R$ (i.e. everywhere within the sphere of radius R). Express your answers in terms of Q, R, and relevant constants. (12%)



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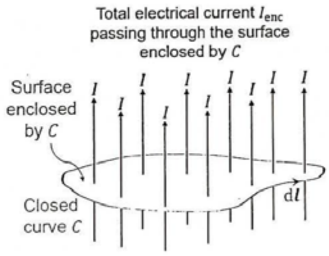
3. Consider an arbitrary closed curve C. Starting with one of the Maxwell's equations in MKS units, derive the following result under steady-state condition:

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 I_{\text{enc}}$$

where the integration is around the closed curve C one time, $\mathrm{d}\boldsymbol{l}$ is a vector that represents an infinitesimal element of the curve C, $\mathbf{B}$ is the magnetic field, I_{enc} is the total electrical current passing through the surface enclosed by C, and μ_0 is the permeability of free space. (10%)

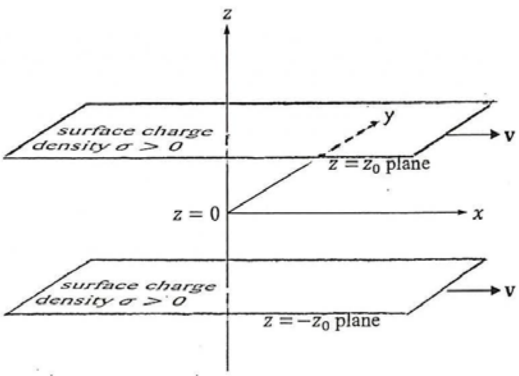


4. There are two identical large thin sheets, each with uniform surface charge density σ, where $\sigma > 0$. The top sheet is in the plane $z = z_0$, and the bottom sheet is in the plane $z = -z_0$, where $z_0 > 0$. As illustrated in the figure below, the two sheets separate the domain of the z coordinate into three regions: $z > z_0$ (above the top sheet), $z_0 > z > -z_0$ (between the two sheets), and $z < -z_0$ (below the bottom sheet). Both sheets are pulled to the $+x$ direction with velocity $\mathbf{v} = v\hat{x}$.

   (a) Find the electric field vector $\mathbf{E}$ as a function of the z coordinate for the three regions specified above. (10%)

   (b) Find the magnetic field vector $\mathbf{B}$ as a function of the z coordinate for the three regions specified above. (10%)

   Show the derivation or provide the reasoning that leads to your results in order to receive full credit.



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5. Three charged particles A, B and C are between two fixed, parallel charged plates of large area. One of the plates has uniform positive charge and the other has uniform negative charge, resulting in a uniform electric field between them. The potential difference between the two plates is 15 volts. We let the electric potential $\phi = 0$ be at the negative plate, and $\phi = 15\,V$ at the positive plate. The three charged particles are momentarily at the midpoint between the two plates, as shown in the figure below. The information about these particles, including their kinetic energy (*KE*) when they are at the midpoint between the plates, is as follows:

Particle A: electric charge $-e$ (e is the elementary charge), mass m_A, $KE = 6$ eV moving toward the negative plate;

Particle B: electric charge $+e$, mass m_B, $KE = 0$ (at rest);

Particle C: electric charge $+e$, mass m_C, $KE = 0$ (at rest).

It is given that $m_A < m_B < m_C$.

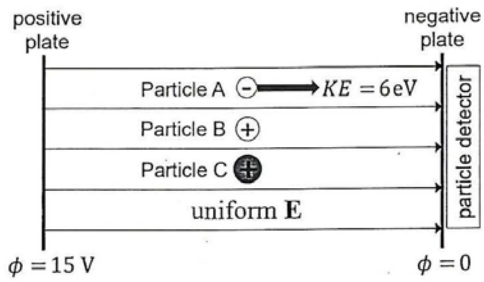
(a) Give the total (kinetic + potential) energy of each charged particle. (3%)

A particle detector is set up at the negative plate to observe all the charged particles that reach there.

(b) Which of the three charged particles will reach the particle detector first? Which one will be the second to reach the particle detector? (2%)

(c) Discuss what will happen to the third charged particle (the particle that is not in your answers in Part (b)) by describing its motion. (3%)

(d) Explain the reason(s) for your answers in Part (b). (2%)



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6. A long pipe running in the z direction has a uniform transverse cross-section that is a square with sides of length 1 meter. The pipe including its walls, which are of negligible thickness, covers the domain $\begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 \end{cases}$ in the x-y plane (the unit of meter has been omitted for simplicity), as shown in the figure below. One particular side of the square pipe, corresponding to the planar surface $y = 1$ in the range $0 \le x \le 1$ (which we denote as $\begin{cases} y = 1 \\ 0 \le x \le 1 \end{cases}$), consists of many individually insulated metal wires running along the pipe such that the electric potential ϕ may vary with the x coordinate. The other three sides of the pipe, which correspond to the planar surfaces $\begin{cases} x = 0 \\ 0 \le y \le 1 \end{cases}$, $\begin{cases} y = 0 \\ 0 \le x \le 1 \end{cases}$, and $\begin{cases} x = 1 \\ 0 \le y \le 1 \end{cases}$, are made entirely of metal and are grounded with electric potential $\phi = 0$. There is no electric charge anywhere.

A spatially varying electric potential represented by the function $\phi(x) = 3\sin(\pi x) + \sin(3\pi x)$ is applied to the entire ungrounded side of the pipe, $\begin{cases} y = 1 \\ 0 \le x \le 1 \end{cases}$. Calculate the electric potential ϕ everywhere inside the pipe, indicating the positions by using the Cartesian coordinates defined in the Problem. (25%)