國立成功大學
111學年度碩士班招生考試試題

編　號：55

系　所：太空與電漿科學研究所

科　目：應用數學

日　期：0219

節　次：第 2 節

備　註：不可使用計算機

國立成功大學 111 學年度碩士班招生考試試題

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

- **Derivation processes have to be given.**

1. (5 %) Please solve the equation: $x^3 = 27$.
2. (5 %) Given $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 1 & 2 \\ 5 & 3 & 1 \end{pmatrix}$, find A^{-1}.
3. (5 %) Given $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, please find all eigenvalues of A and their respective eigenvectors.
4. (5 %) Given $y^3 + 3y^2 - 6y + 3x^2 + 5 = 0$, please find $\frac{dy}{dx}$.
5. (5 %) Please calculate the following integration:

$$\int_0^\infty e^{-x^2}\, dx = ?\ .$$

6. (5 %) From the Leibniz integral rule, we have known that

$$\frac{\partial}{\partial t}\left(\int_{a(t)}^{b(t)} f(x,t)\, dx\right) = f(b(t),t)\frac{db(t)}{dt} - f(a(t),t)\frac{da(t)}{dt} + \int_{a(t)}^{b(t)} \frac{\partial f(x,t)}{\partial t} dx\ .$$

Please answer the following question:

$$\frac{\partial}{\partial t}\left(\int_{\sin(t)}^{\cos(t)} e^{-x^2} dx\right) = ?\ .$$

7. (15 % in total) For a given scalar function in space $V(x,y,z) = e^x \sin(y) \ln(z)$, please calculate:
   (a) (5 %)$\nabla V = ?$;
   (b) (5 %)$\nabla \cdot (\nabla V) = ?$;
   (c) (5 %)$\nabla \times (\nabla V) = ?$.
8. (10 %) For a given vector $\vec{V} = (2xy + z^2)\,\hat{x} + 3xz\hat{y}$, please calculate the counterclockwise line integral of a rectangular in the range of $0 \le x \le 2$ and $0 \le y \le 3$ on the x-y plane:

$$\oint \vec{V} \cdot d\vec{l} = ?\ .$$

9. (15 % in total) We have known that the Fourier transform of a function $g(t)$ is

$$G(\omega) \equiv F\{g(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} g(t)\, dt .$$

On the other hand, the inverse Fourier transform of the function $G(\omega)$ is

$$g(t) \equiv F^{-1}\{G(\omega)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t} G(\omega)\, d\omega .$$

Let function

$$X(\omega) = \begin{cases} 0 & \text{for } |\omega| > \omega_f \\ 1 & \text{for } |\omega| \le \omega_f \end{cases}$$

and function

$$y(t) = \cos(\omega_o t) .$$

Please show that:

(a) (8 %) $F^{-1}\{F\{y(t)\} \times X(\omega)\} = 0$ for $\omega_o > \omega_f$.

(b) (7 %) $F^{-1}\{F\{y(t)\} \times X(\omega)\} = y(t)$ for $\omega_o < \omega_f$.

10. (10 %) Please solve the following ordinary differential equation with the initial condition $x(0) = 0$

$$\frac{dx(t)}{dt} + Ax(t) = t$$

where A is a constant.

11. (20 % in total) A function $U(t,x)$ is the solution of the following diffusion equation

$$\frac{\partial U(t,x)}{\partial t} - \frac{\partial^2 U(t,x)}{\partial x^2} = 0$$

in a domain of $0 \le x \le \pi$. The boundary condition of the solution is $U(t,0) = U(t,\pi) = 0$. The initial condition is $U(0,x) = \sin(x)$. Please

(a) (10 %) obtain the function $U(t,x)$;

(b) (5 %) draw $U(0,x)$, $U(1,x)$, and $U(\infty,x)$ versus x in the same plot;

(c) (5 %) draw $U\left(t, \frac{\pi}{2}\right)$ versus t.