

國立成功大學

113學年度碩士班招生考試試題

編 號：55

系 所：太空與電漿科學研究所

科 目：應用數學

日 期：0201

節 次：第 2 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

- Derivation processes have to be given.

1. (5 %) Please give the result of the summation: $\sum_{i=1}^{10} 1 = ?$

2. (5 %) Given $A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 11/7 & -1/7 \\ 0 & -1/7 & 2/7 \end{pmatrix}$, find A^{-1} .

3. (10 %) Given $A = \begin{pmatrix} 5 & 0 & \sqrt{3} \\ 0 & \sqrt{3} & 0 \\ \sqrt{3} & 0 & 3 \end{pmatrix}$, please find all eigenvalues of A and their respective eigenvectors.

4. (10 %) Please calculate the following integration:

$$\int_{-\infty}^{\infty} \frac{dx}{4+x^2} = ?$$

5. (9 %) For a given vector $\vec{V} = (x^2 + 3y)\hat{x} + (2x + y^2)\hat{y} + (3x^2 + 2y^2 + 3z^2)\hat{z}$, please calculate the counterclockwise line integral along the circumference of an ellipse $\frac{x^2}{2} + \frac{y^2}{3} = 1$:

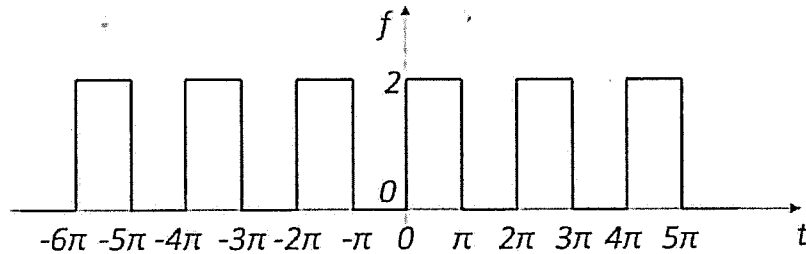
$$\oint \vec{V} \cdot d\vec{l} = ?$$

6. (9 %) Let the vector $\vec{V} = (x + 3y^2 + 2z^3)\hat{x} + (2x^2 + 2y + z^2)\hat{y} + (3x^3 + y^3 + 3z)\hat{z}$, please calculate the surface integral on the surface of a sphere with a radius of 3:

$$\int \vec{V} \cdot d\vec{A} = ?$$

where $d\vec{A}$ is pointing out of the spherical surface.

7. (12 %) Please find the Fourier series of a square function shown in the following figure.



8. (15 %) Please solve the following ordinary differential equation with the initial condition $y(0) = 0$

$$\frac{dy(x)}{dx} + \alpha y(x) = \cos(\omega x)$$

where α and ω are constants.

9. (15 % in total) Following is a second-order ordinary differential equation with the initial condition $y(0) = 0$ and $y'(0) = \alpha$

$$m \frac{d^2 y(t)}{dt^2} + ky(t) = 0$$

where m and k are constants. Please

- (7 %) apply the Laplace transform to the equation,
- (4 %) solve the equation in the Laplace domain,
- (4 %) show the solution in the time domain.

10. (10 %) A function $U(x, y)$ is the solution of the following Laplace equation

$$\frac{\partial^2 U(x, y)}{\partial x^2} - \frac{\partial^2 U(x, y)}{\partial y^2} = 0$$

in a rectangular of $0 < x < 3$ and $0 < y < 2$. The boundary condition of the solution is $U(0, y) = U(x, 2) = U(3, y) = 0$ and $U(x, 0) = 10 \sin(\pi x/3)$. Please solve the equation and obtain the function $U(x, y)$.