

# 國立成功大學

## 115學年度碩士班招生考試試題

編 號：46

系 所：太空與電漿科學研究所

科 目：應用數學

日 期：0204

節 次：第 2 節

注 意：1. 不可使用計算機  
2. 請於答案卷(卡)作答，於  
試題上作答，不予計分。

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- **Answers must be described logically and straightforwardly so that readers can follow easily.**
- **Calculation processes have to be described.**

### I. Ordinary differential equations (20 percent):

Answer to the following questions:

- (i) Find the general solution of the following ordinary differential equation

$$dN/dt = -N/\tau, N(t=0) = N_0. \text{ (3 percent)}$$

- (ii) Find the general solution of the following ordinary differential equation by separation of variables:

$$y' = dy/dx = (x-1)(y+1). \text{ (5 percent)}$$

- (iii) Find the general solution of the following inhomogeneous ordinary differential equation:

$$y' + 3xy = dy/dx + 3xy = \cos(2x). \text{ (12 percent)}$$

### II. Fourier transformation (20 percent):

Calculate the Fourier transform of the following functions

(i)  $f(x) = \begin{cases} e^{-ax} & (x \geq 0) \\ 0 & (x < 0) \end{cases}, a > 0. \text{ (10 percent)}$

(ii)  $f(x) = \exp(-\frac{x^2}{a^2}), a > 0. \text{ (Hint: } \int_{-\infty}^{\infty} \exp(-ax^2) dx = \sqrt{\frac{\pi}{a}} \text{ ) (10 percent)}$

### III. Vector operations (23 percent):

Answer to the following questions:

- (i) Calculate  $\nabla f(r)$ ,  $r \equiv \sqrt{x^2 + y^2 + z^2}$ ,  $\nabla = i\partial/\partial x + j\partial/\partial y + k\partial/\partial z$ .  $i, j$  and  $k$  are basis vectors in directions of  $x, y$  and  $z$ , respectively. (5 percent)

- (ii) Find  $\nabla(1/r)$  ( $r \neq 0$ ). (5 percent)

- (iii) Calculate  $\nabla \cdot (\nabla(1/r))$  ( $r \neq 0$  and  $r=0$ ). Hint: Use the Gauss theorem

$$\int_{\Omega} \text{div} f dV = \int_{\Sigma} f \cdot n dS \text{ and Dirac's delta function } \delta(r), \int \delta(r-r_0) f(r) dV = f(r_0). \text{ (5 percent)}$$

- (iv) Calculate  $\nabla \times (f(r)\vec{r})$ . (8 percent)

**IV. Eigen values, vectors, and diagonalization (22 percent):**

Find the eigenvalues and normalized three eigen vectors of the matrix  $T$  shown below (2 percent each, total 12 percent). From those, find a matrix  $B$ , that can diagonalize the matrix  $T$  by a similarity transformation, and show the diagonalized matrix  $D = B^{-1}TB$  (8 + 2 percent).

$$T = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

**V. Find the following values of the complex logarithm (15 percent):**

(1)  $\log 2$ , (5 percent)    (2)  $\log(-2)$  (5 percent)    (3)  $\log(1-i)$  (5 percent)