(1) Scratch a simple diagram to explain the geometrical meanings of the following quantities: (a) $\vec{A}\cdot(\vec{B}\times\vec{C})$, (b) $\nabla\varphi$, (c) $\nabla\cdot\vec{A}$, (d) $\nabla\times\vec{A}$. (15%)

(2) Evaluate $\iint \nabla\times(y\hat{i}+3\hat{j}+5\hat{k})\cdot\hat{n}d\sigma$, where σ is the surface in the first octant made up of part of the plane x+2y+3z=6, and triangles in the (x, z) and (y, z) planes. (10%)

(3) (a) Solve $dN/dt + \alpha N = e^{-\beta t}$, where α, β are constants. (5%) (b) Find the general solution of the differential equation $d^2x/dt^2 + 5\,dx/dt + 4x = 2\cos(2t)$, and give some discussions on the physical meaning of the complementary function and the particular solution. (10%)

(4) Find the Fourier series representation of function

$$f(t)=\begin{cases} 0, & -\pi \le \omega t < 0 \\ \sin\omega t, & 0 \le \omega t < \pi \end{cases} . (10\%)$$

(5) (a) Prove the Cauchy's integral formula $\oint_C \frac{f(z)dz}{(z-a)} = 2\pi i\, f(a)$ by using Cauchy's theorem $\oint_C g(z)dz = 0$, where *f(z)* and *g(z)* are analytical function inside the contour *C* (8%) (b)Evaluate the definite integral $\int_0^\infty \frac{dx}{(4x^2+1)^3}$. (7%)

(6) Find the eigenvalues and eigenvectors of the matrix $M=\begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$. (15%)

(7) In the initial steady state of an infinite slab of the thickness d, the face x=0 is at 0 ℃ and the face x=d is at T_0. From t=0 on, the x=0 face is held at T_0 and the x=d face at 0℃. Find the temperature distribution at time t, T(x,t). (Note: T(x,t) obeys the diffusing equation $\nabla^2 T(x,t) = \frac{1}{\alpha^2}\frac{\partial T(x,t)}{\partial t}$.)(20%)