1. You are asked to solve the one dimensional potential distribution from Laplace equation with the method of relaxation. The boundary conditions are V(x=1) = V(1) = 4, V(5) = 0. Calculate V(2), V(3), and V(4) up to five steps. (15%)

2. Consider a uniform linear dielectric material of susceptibility χ_e fills the entire region below the plane $z = 0$. Calculate the distribution and the total of the surface bound charge induced by a charge q located at $z = a$. (20%)

3. Find the cyclotron frequency of an electron located at half the radius R of a very long cylindrical solenoid, consisting of n closely wound turns per unit length and carrying a steady current I. (15%)

4. Consider the wave packet that takes the form

$$\Psi(x,t) = \int_{-\infty}^{\infty} dkA(k)e^{i(kx-\omega t)} \qquad (4.1)$$

Suppose $A(k)$ is given by the normal distribution

$$A(k) = e^{-\alpha(k-k_0)^2} \qquad (4.2)$$

i.e., the wave centers around the wave number k_0. (20%)

a. Show that the wave has a spread in the k-spectrum with the width given by

$$\Delta k = 1/\sqrt{2\alpha}. \qquad (4.3)$$

b. Carry out the integral in Eq(4.1) to find $\varphi(x)$ where $\Psi(x,t) = \varphi(x)e^{ik_0x-i\omega t}$.

c. From b, calculate Δx the spatial width of the wave packet.

d. Find the product of Δk and Δx.

e. Explain the physical meaning of the result in d.

（背面仍有題目,請繼續作答）

5. Consider a lattice of immobile hydrogen nucleons. The electrons are treated as the electron gas. The atomic density is n, and the electron density n_e is a function of space and time but $\langle n_e \rangle = n$ after taking the space and time average as denoted by the angle bracket. (15%)

a. Write down the Poisson's equation that relates the electric field to the charge density.

b. Write down the equation of motion for the electrons.

c. Given the continuity equation of charged particles

$$\partial n / \partial t + \nabla \cdot n\vec{v} = 0 \qquad (5.1)$$

Linearize the Poisson's equation, the continuity equation and the equation of motion. Assume a plane wave solution for all the perturbations, namely, $\delta n, \delta\vec{v}, \delta\vec{E} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, find the dispersion relation for the wave number $\vec{k}$ and wave frequency ω.

d. Explain the physical meaning of this dispersion relation.

6. The magnetic flux is defined as

$$\Phi \equiv \int \vec{B} \cdot d\vec{S}. \qquad (6.1)$$

The vector potential $\vec{A}$ is defined from

$$\vec{B} = \nabla \times \vec{A}. \qquad (6.2)$$

An electron has the particle momentum $\vec{p}$. (15%)

a. Solve $\vec{A}$ for a constant magnetic field $\vec{B} = B_0 \hat{e}_z$ pointing in the z-direction.

b. Write down the generalized momentum $\vec{P}$.

c. Show that

$$\Phi = \int_S \vec{A} \cdot d\vec{l}, \qquad (6.3)$$

where the integral is taken along the particle trajectory of a closed surface S.

d. Show that Φ is a constant of the motion.