※ 考生請注意：本試題 □可 ☑不可 使用計算機

**Show your calculation steps carefully. A correct answer, unsupported by calculations, explanation, or algebraic work will not receive credit; partial credit will be given for correct steps.**

**1.** (15%) Solve the following ODEs:

(a) (7%) $y' = y + xy^2$

(b) (8%) $(2xy^3 + y^4)\,dx \; + \; (xy^3\text{-}2)\,dy = 0$

**2.** (15%) Solve the following initial value problem by the Laplace Transform:

$$y' = 1 - H(t-1)\,, \;\; y(0) = 0$$

Where $H(t)$ represents the Heaviside function and is defined to be

$$H(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0 \end{cases}$$

[Hint: Shift rule of Laplace Transform $\mathcal{L}[H(t-a)g(t-a)](s) = e^{-as}\mathcal{L}[g](s)$ may be helpful to you]

**3.** (10%) Solve

$$y'' - xy' - y = 0$$

at the point $x = 0$ using the power series method and find the recurrence relation and the first 4 terms of the two linearly independent solutions.

**4.** (20%) The cosine of a matrix $A$ is defined by copying the series for $\cos(x)$ (which always converges):

$$\cos A = I - \frac{1}{2!}A^2 + \frac{1}{4!}A^4 - \cdots$$

(a) (10%) Suppose $x$ is an eigenvector of matrix $A$ and $Ax = \lambda x$. Please show that $x$ is also an eigenvector of $\cos A$ and find its corresponding eigenvalue.

(b) (10%) The eigenvectors of matrix $A = \frac{\pi}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ are (1,1) and (1,-1). From the eigenvectors and eigenvalues, find the matrix $\cos A$.

(背面仍有題目,請繼續作答)

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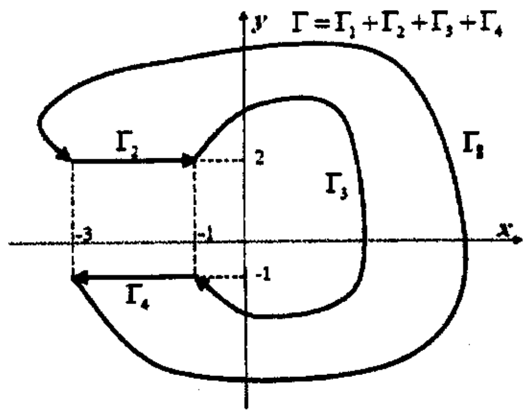
**5.** (20%) The figure shows a positively oriented closed curve $\Gamma$ composed of four separate curves $\Gamma_i$, ($i$=1,2,3,4). In $x$-$y$ plane, a vector function $\vec{F}$ is defined as:

$$\vec{F} = (y^5 - y^3)\,\vec{i} + (5xy^4 - 4y - 3xy^2)\,\vec{j}$$




(a) (10%) Evaluate the value of integral

$$I = \oint_{\Gamma} \vec{F} \cdot d\vec{r}$$

(b) (10%) If the integral $\int_{\Gamma_3} \vec{F} \cdot d\vec{r} = 10$, evaluate the value of the integral $S = \int_{\Gamma_1} \vec{F} \cdot d\vec{r}$.

**6.** (a) (10%) Given $f(t) = 1 - |t/2|$, $-2 \le t \le 2$ and zero elsewhere, show that the Fourier transform of

$f(t)$ is $F(\omega) = \sqrt{\dfrac{2}{\pi}}\left(\dfrac{\sin\omega}{\omega}\right)^2$

(b) (5%) Prove Parseval relation: $\int_{-\infty}^{\infty} F(\omega) G^*(\omega)\, d\omega = \int_{-\infty}^{\infty} f(t) g^*(t)\, dt$

Where $F(\omega)$ and $G(\omega)$ are the Fourier transforms of $f(t)$ and $g(t)$ respectively. * denotes the complex conjugation.

[Hint: the expression of Dirac delta function $\delta(t-x) = \dfrac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega(t-x)} d\omega$ may be helpful.]

(c) (5%) Evaluate $S = \int_{-\infty}^{\infty} \left(\dfrac{\sin t}{t}\right)^4 dt$ by using the Parseval relation.