系所組別： 太空天文與電漿科學研究所
考試科目： 電磁學
考試日期：0308，節次：2

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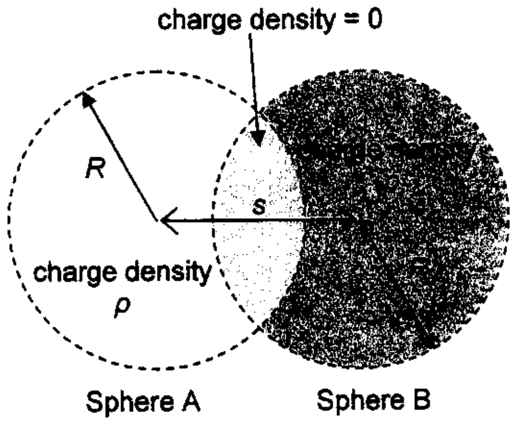
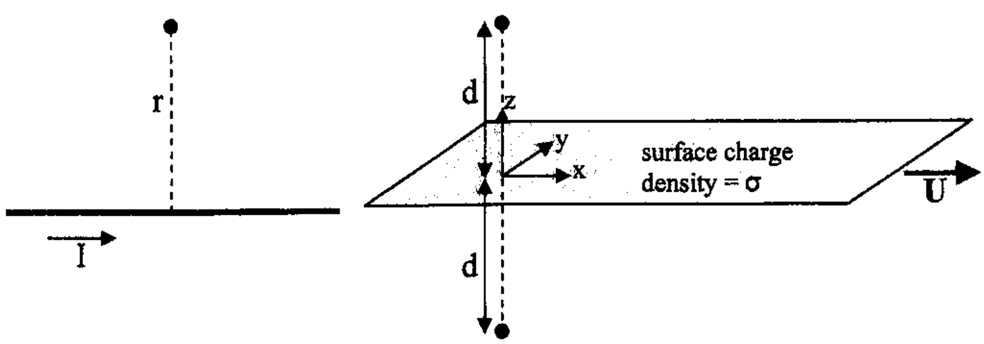
1. A structure of non-conducting material is in the form of two overlapping spheres as shown in the figure below. Both spheres are of radius R and their centers are a distance s apart, where $R < s < 2R$. The non-overlapped regions of Sphere A and Sphere B have uniform charged density ρ and $-\rho$, respectively, while their overlapped region has zero charge density. Show that the electric field within the overlapped region of the two spheres is constant, and find its value. (20%)



2. (a) Find the magnetic field at a perpendicular distance r away from an infinite straight wire carrying current I. (5%)

   (b) An infinite thin sheet with uniform surface charge density σ occupies the entire $z = 0$ plane. The sheet is moving with a constant velocity $\mathbf{U} = U\hat{x}$. Determine the magnetic field at the points $(x, y, z) = (0, 0, d)$ and $(0, 0, -d)$ with $d > 0$ by constructing an integral based on the result in (a). Please indicate clearly the direction of the magnetic field at both points. Then determine the magnetic field everywhere except at the $z = 0$ plane. (10%)



(背面仍有題目,請繼續作答)

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3. Solve for the magnetic field everywhere except at the $z = 0$ plane due to the moving charge sheet in Problem 2(b) again; this time to apply the Lorentz transformation of electric and magnetic fields to a moving frame. The transformation rules are as follows:

$$E_{\parallel}' = E_{\parallel}\ ; \quad \frac{\mathbf{E}_{\perp}'}{c} = \gamma\left(\frac{\mathbf{E}_{\perp}}{c} + \boldsymbol{\beta}\times\mathbf{B}_{\perp}\right)\ ;$$

$$B_{\parallel}' = B_{\parallel}\ ; \quad \mathbf{B}_{\perp}' = \gamma\left(\mathbf{B}_{\perp} - \boldsymbol{\beta}\times\frac{\mathbf{E}_{\perp}}{c}\right),$$

where the primed quantities are fields in frame S' moving with constant velocity $\mathbf{v} = c\boldsymbol{\beta}$ relative to frame S, for which the E and B fields are represented by the unprimed quantities, the $\parallel$ and $\perp$ subscripts refer to directions parallel and perpendicular to $\mathbf{v}$, respectively, c is the speed of light, and $\gamma = \left(1-\beta^2\right)^{-1/2}$. Assume that $U^2 \ll c^2$ in your calculation to simplify the solutions. (15%)

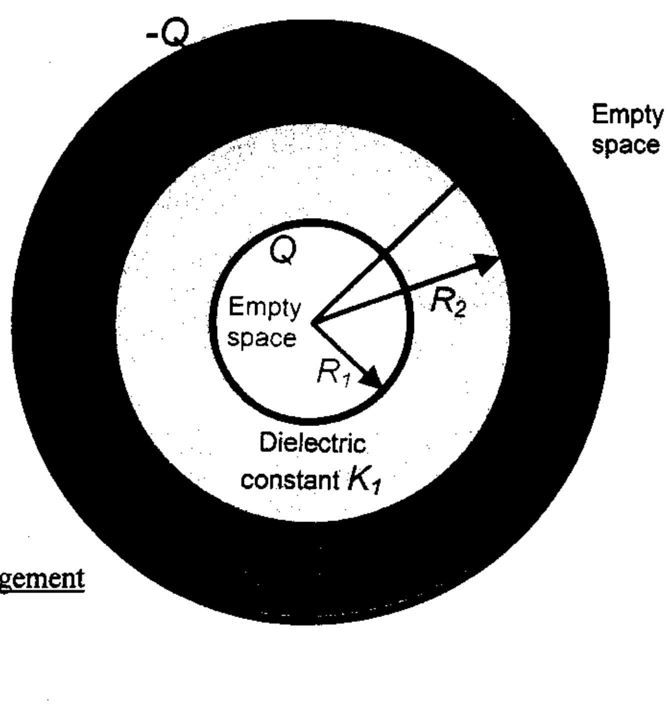
4. Consider the spherical arrangement shown in the figure below: empty space from $0 < r < R_1$; linear dielectric of constant $K_1 > 1$ between $R_1 < r < R_2$; linear dielectric of constant $K_2 > 1$ between $R_2 < r < R_3$; and empty space for $r > R_3$.

Free charge of amount Q is uniformly distributed over the inner surface at $r = R_1$, and free charge of amount $-Q$ is uniformly distributed over the outer surface at $r = R_3$. There are no other free charges anywhere else.

(a) Determine the vector $\mathbf{E}$ and $\mathbf{D}$ fields everywhere. (8%)

(b) Determine the volume bound charge density ρ_b and surface bound charge density σ_b everywhere. (8%)

(c) What is the capacitance of the spherical arrangement? (4%)



Spherical arrangement in Problem 4.

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5. Consider a large region with no electric field and a uniform magnetic field $\mathbf{B} = B\hat{z}$. The equation of motion for a particle of electric charge q and mass m is:

$$m\frac{d^2\mathbf{v}}{dt^2} = q\mathbf{v}\times\mathbf{B} \ . \qquad (5.1)$$

(a) Solve Eq. (5.1) to obtain the velocity $\mathbf{v} = (v_x, v_y, v_z)$ and position $\mathbf{x} = (x, y, z)$ of the particle as a function of time t, using the following initial conditions at $t = 0$: $\mathbf{x} = (0,0,0)$ and $\mathbf{v} = (v_\perp, 0, 0)$. (10%)

(b) Give an expression for the cyclotron frequency Ω of the particle by identifying its periodic motion. (3%)

(c) From the trajectory of the particle, identify the Lamour radius r_L. Express r_L in terms of Ω and other quantities or parameters given in the problem. (2%)

6. Fast electromagnetic waves in plasma, under the assumption that they propagate as plane waves (i.e., $\mathbf{E}(\mathbf{x},t)$ and $\mathbf{B}(\mathbf{x},t) \sim \exp[i(\mathbf{k}\cdot\mathbf{x} - \omega t)]$), have the following dispersion relation:

$$\omega^2 = \omega_{pe}^2 + c^2k^2 \ , \qquad (6.1)$$

where ω is the wave frequency, $\mathbf{k}$ is the wave vector, $k = |\mathbf{k}|$, ω_{pe} is the electron plasma frequency, and c is the speed of light.

(a) Based on Eq. (6.1), determine the phase velocity v_{ph} and the group velocity v_{gr} of the waves without using k in your answers. (12%)

(b) Show that $v_{gr} < c$. (3%)