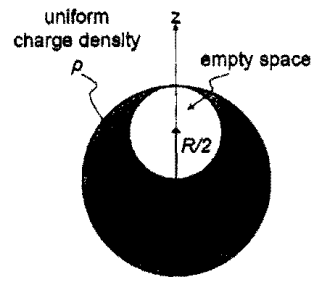
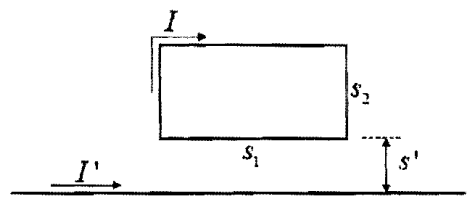
※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. Consider the structure shown in the figure on the right. It is a sphere of radius R but with empty space carved out in the shape of a small sphere of radius $R/2$. The distance between the center of the small empty sphere and that of the large sphere is $R/2$. The structure is made of non-conducting material of uniform charge density ρ. Suppose we choose a coordinate system such that the center of the large sphere is at the origin and the center of the small empty sphere is at $(R/2)\hat{z}$. Find the electric field vector everywhere within the spherical empty space. (20%)



Large sphere with small spherical empty space

2. An infinite straight wire and a rectangular wire loop, lying on the same plane, are carrying steady current I' and I respectively. As shown in the figure on the right, the length and width of the loop are s_1 and s_2 respectively, with its length running parallel to the infinite wire, separated by a distance s'. Find the force on the rectangular loop due to the infinite straight wire. (10%)



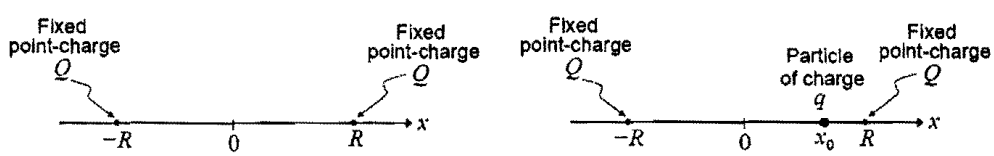
3. An infinite, one-dimensional grid is along the x-axis. It has two fixed point-charges of electric charge $Q>0$, located at $x=R$ and $x=-R$ (see left figure below).

   (a) Find the electric potential $V=V(x)$ everywhere along the one-dimensional grid, and sketch $V(x)$ vs. x. (5%)

   A particle of mass m and electric charge $q>0$ is now placed in the grid initially at rest at $x=x_0$, where $0<x_0<R$ (see right figure below). The charged particle moves under the influence of the electric field without friction, but its motion is limited to the direction along the grid.

   (b) Give the range of locations in the grid that is accessible to the charged particle. (5%)

   (c) Determine the maximum possible speed of the charged particle as it moves along the grid, as well as the location where this maximum speed is attained. (10%)



(背面仍有題目，請繼續作答)

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4. A plane electromagnetic wave propagates in z direction in a uniform isotropic medium with permittivity ε, permeability μ and conductivity σ.

(i) Show a relation between real angular frequency ω and complex wave number k when the x component of the electric field can be represented as

$$E(z, t) = E_0 \exp[i(\omega t - kz)] \quad \text{(8 percent)}$$

(ii) Derive the x component of the magnetic field. (6 percent)

(iii) Express Re[k] and Im [k] by using ε,μ and σ. (6 percent)

5. Magnetic field $\boldsymbol{B}$ and electric field $\boldsymbol{E}$ induced by electric dipole (whose moment is $\boldsymbol{P}$) are written as

$$\boldsymbol{B} = \frac{\mu_0}{4\pi c r} \ddot{\boldsymbol{P}}(t') \times \boldsymbol{n},$$

$$\boldsymbol{E} = \frac{\mu_0}{4\pi r} \left(\ddot{\boldsymbol{P}}(t') \times \boldsymbol{n} \right) \times \boldsymbol{n},$$

respectively. Here $t'=t-r/c$ and $\boldsymbol{n} = \boldsymbol{r}/r$, where $\boldsymbol{r}$ and c are position vector of the observation location and speed of light in vacuum, respectively. Give expressions of pointing vector $\boldsymbol{S}$ and energy I emitted from the electric dipole per unit time using $\boldsymbol{P}$, μ_0, c, r and $\boldsymbol{n}$. Use the vector identity $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \cdot \boldsymbol{C})\boldsymbol{B} - \boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B})$, if necessary. (10 and 5 percent, respectively)

When an electric dipole (whose magnitude of electric dipole moment is P_0) rotates on x-y plane about z axis with angular frequency of ω, calculate the energy emitted from the dipole per unit time. Hint: Integrate the $\boldsymbol{S}$ on a sphere surface. (15 percent)