1. An elastic material fills a cavity with sides 2a and height L in a rigid block, as shown in Fig. 1. A rigid cap is placed on the top of the elastic material, and a compressive force F is applied to the cap at the same time as the temperature is increased. (25%)

(a) Find the displacement of the cap

$$\frac{C}{L} = \frac{1}{E}\,\frac{1+\nu}{1-\nu}\left[-(1-2\nu)\frac{F}{4a^2} + F\alpha\Delta T\right]$$

(b) The force required to keep the cap in its initial position c=0, i.e.,

$$\frac{F}{4a^2} = 3B\alpha\Delta T$$

Where B is the bulk modulus.

2. A steel plate is clad with a thin layer of soft aluminum on both sides, as shown in Fig. 2 . The temperature of the cladded system is raised by an amount ΔT. Because the cladding is thin and of a lesser stiffness than the steel, the stresses in the plate are negligible. Stresses do arise in the cladding, however, from the mismatch of the coefficients of thermal expansion and the requirement that the axial strains in the plate and cladding be equal. (25%)

(a) Show that the normal stresses σ_x and σ_y in the cladding are

$$\sigma_x = \sigma_y = \frac{E_c(\alpha_p - \alpha_c)}{1-\nu_c}\Delta T$$

(c) If $\alpha_c = 12\times10^{-6}/{}^\circ F$, $\alpha_p = 6\times10^{-6}/{}^\circ F$, $\nu_c = 0.33$, $E_c = 11\times10^{6}\,psi$;

And the yield stress of the aluminum is Y=5ksi, find the change in temperature to cause initial yielding by using the maximum shear stress criterion.

3. A machine part consists of two stiff rigid metal pieces AB and CD separated by a flexible strip BC of bending modulus EI, shown in Fig. 3. (25%)

(a) Take parts AB and CD to be completely rigid in bending, and calculate the moment M_A applied at section A which is required to keep AB in a vertical position.

(b) Find total vertical deflection v_A of section A.

（背面仍有題目,請繼續作答）

4. The concept behind an atomic force microscope used to scan the atomic structure of the surfaces is to fabricate a cantilever beam whose spring constant is smaller than the equivalent spring between atomics, which is the order of 10N/m. Calculate the equivalent spring constant for the cantilever beam shown in Fig. 4. It has been claimed that a cantilever spring made from a piece of aluminum foil with the L and W dimensions shown has a spring constant of the order 1N/m. Is this true?
E = 70Gpa　　(25%)

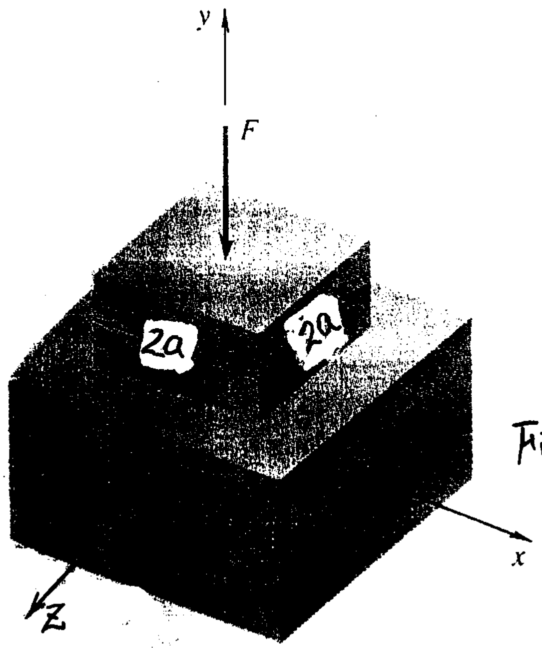
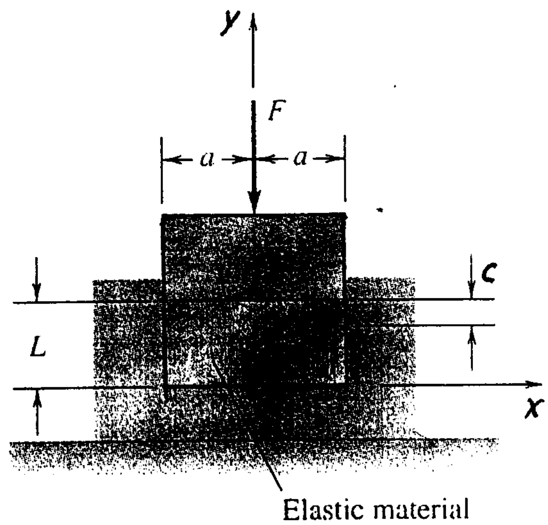


Fig. 1



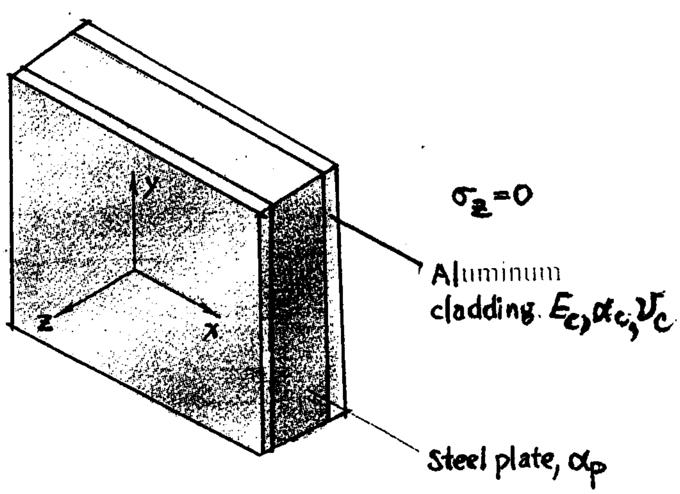


Fig 2.

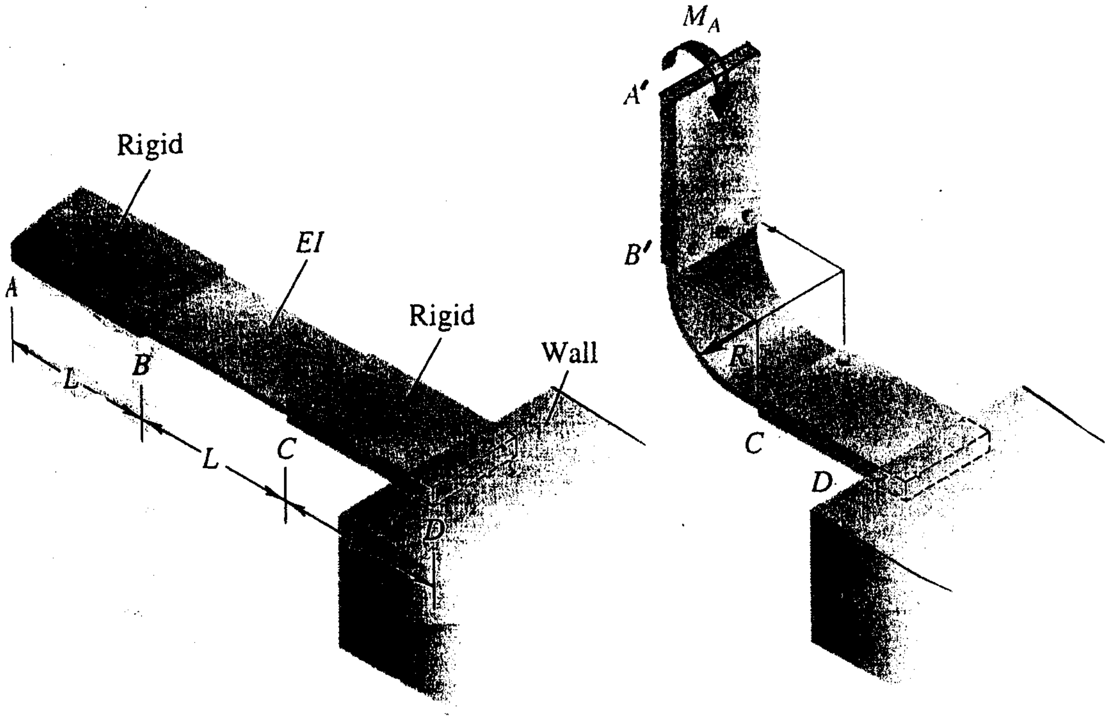
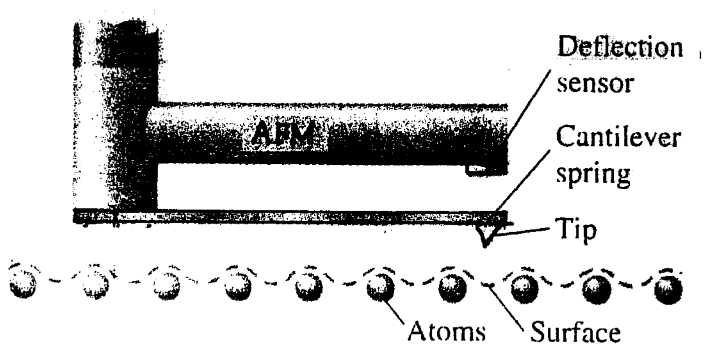
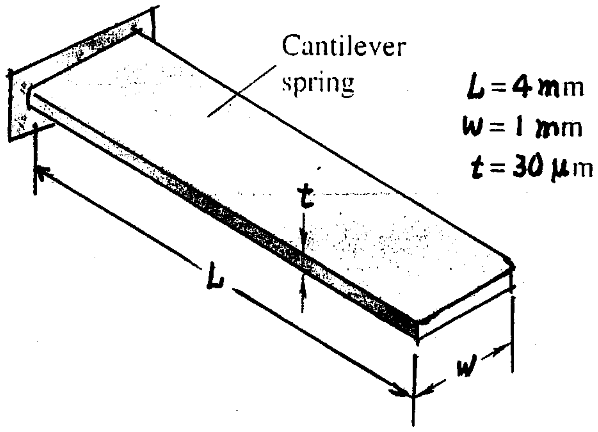

Rigid
EI
Rigid
Wall
A
B
C
D
L
L
M_A
A′
B′
R
C
D

Fig. 3


AFM
Deflection sensor
Cantilever spring
Tip
Atoms
Surface


Cantilever spring
L = 4 mm
w = 1 mm
t = 30 μm
t
L
w