

1. 試解常微分方程式

(25%)

$$\frac{d^2x}{dt^2} + \omega_0^2 x = A \sin \omega t$$

在此 ω_0, ω, A 皆為正常數。並討論 $\omega_0 \neq \omega$ 及 $\omega_0 = \omega$ 兩種情況之通解

2. 試解一維熱傳導偏微分方程式

(25%)

$$\frac{\partial T}{\partial t} = c^2 \frac{\partial^2 T}{\partial x^2}$$

在此 $t \geq 0$. $0 < x < l$.其起始條件: $T(x, 0) = f(x)$ 邊界條件: $T(0, t) = 0,$ $T(l, t) = 0$ } $t \geq 0$ 在此 $f(x)$ 假設為已知函數。

3. Apply the convolution theorem and Laplace transform to
(25%) determine f such that

$$f(t) = 2t^2 + \int_0^t f(t-\alpha) e^{-\alpha} d\alpha$$

4. (a) Prove the Cauchy-Schwarz inequality
(25%)

$$|F \cdot G| \leq \|F\| \|G\|$$

where F and G are vectors in \mathbb{R}^n

(b) Give at least one application example for the above inequality