

1. 試答下列問題:

a) 試列出 Sturm-Liouville equation 之通式. (8%)  
 並說明何謂 Sturm-Liouville problem.

b) Sturm-Liouville problem 之解具何種特性? (12%)

2. 試解  $y'' + y = \sec x$ . (15%)

3. 試解波動方程式 (15%)

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad (0 < x < L, t > 0)$$

其起始條件:

$$\left. \begin{aligned} y(x, 0) &= f(x) \\ \frac{\partial y}{\partial t} \Big|_{(x, 0)} &= g(x) \end{aligned} \right\} 0 < x < L$$

其邊界條件:  $y(0, t) = y(L, t) = 0 \quad t > 0$

\* 註:  $f(x), g(x)$  為已知函數.

4. (a) What is the general procedure for solving an ordinary differential equation with Laplace Transforms? (10%)
- (b) Please solve the following ordinary differential equation with Laplace Transforms. (15%)

$$\frac{dx}{dt} + \frac{dy}{dt} + x = -e^{-t}$$

$$\frac{dx}{dt} + 2\frac{dy}{dt} + 2x + 2y = 0$$

Initial conditions:

$$x(0) = -1, \quad y(0) = \pm 1$$

5. (a) What is Green's Theorem in vector analysis? (10%)
- (c) Let  $C$  be a positively oriented simple closed path with interior  $D$ . Show that the centroid of  $D$  is  $(\bar{x}, \bar{y})$ , where (15%)

$$\bar{x} = \frac{1}{2A} \oint_C x^2 dy$$

$$\bar{y} = \frac{1}{2A} \oint_C y^2 dx$$