(每題 20 分)

1. Please find a general solution of

$$y' = Ay + g = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} y + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}$$

2. Show that

$$\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$$

3. One solution of Bessel's equation,

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(x^2 - p^2\right) y = 0$$

is called $J_p(x)$.　Show that a second solution is of the form

$$J_p(x) \int \frac{dx}{x\left[J_p(x)\right]^2}$$

4. Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

5. A sinusoidal voltage $E \sin wt$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave.　Find the Fourier series of the resulting periodic function.

$$u(t) = \begin{cases} 0 & if \quad -L < t < 0 \\ E \sin wt & if \quad 0 < t < L \end{cases} \qquad p = 2L = \frac{2\pi}{\omega}, \ L = \frac{\pi}{\omega}$$