

本試題共分六大科目: (一) 工程數學 (二) 熱工學 (三) 機械設計 (四) 材料力學 (五) 機械製造 (六) 自動控制。每一科目含 2 題, 共計 12 題。考生可任選四題作答, 總分為 100 分。

(一) 工程數學

1. Solve the following problem (25%)

$$\frac{d^2 y}{dt^2} - 3 \frac{dy}{dt} + 2y = 2 + \delta(t - 5)$$

with  $\frac{dy}{dt}(0) = 0$  and  $y(0) = 0$

where  $\delta(t - 5)$  is known as the Dirac delta function.

2. Find a normal vector  $\bar{n}$  and the equation of the tangent plane for a given surface  $S: z = x^2 + y^2$  and a point  $P(2, 1, 5)$ . (25%)

(二) 熱工學

3. What is the COP (coefficient of performance) of a refrigerator? Calculate the value of COP for a Carnot cycle refrigerator operating between temperatures of  $-20^\circ C$  and  $25^\circ C$ . Explain the difference in the viewpoints taken for heat pumps and refrigerators. (25%)
4. A body of air is heated at constant pressure. What fraction of the entropy added remains in the body? Assume that the heat capacities of air are constant. Then, what fraction of the energy added as a result of heating remains there? (25%)

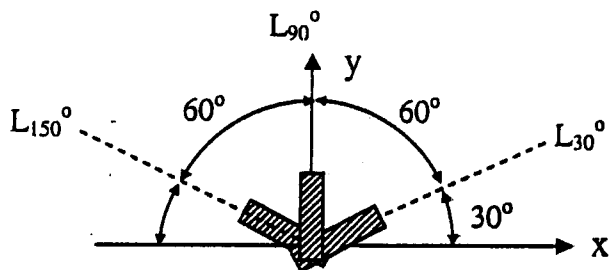
(三) 機械設計

5. (a) What is the criteria to distinguish the long column, and short column (10%).  
(b) What are the best theories to predict critical loads for long columns and short columns? (15%)
6. (a) Draw a Modified Goodman Diagram and explain what is the safe region to predict fatigue failure. (15%)  
(b) Explain the Miner's cumulative rule for fatigue damage. (10%)

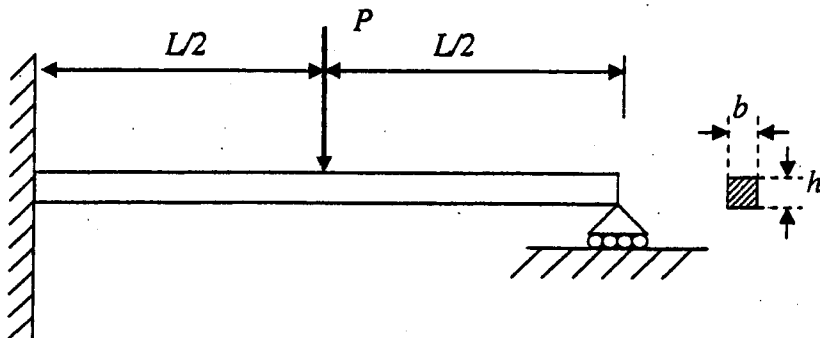
(背面仍有題目, 請繼續作答)

(四) 材料力學

7. As shown in the figure, a strain gage rosette is bonded to a free surface for measuring relative elongation along three different directions,  $30^\circ$ ,  $90^\circ$ , and  $150^\circ$ . If the measured elongation along the three directions are,  $L_{30^\circ} = 0.003$ ,  $L_{90^\circ} = -0.003$ , and  $L_{150^\circ} = 0.006$ , please determine the strain components  $\epsilon_{xx}$ ,  $\epsilon_{yy}$ , and  $\gamma_{xy}$ . (25%)



8. A beam is subjected to a loading  $P$  as shown in figure. Please determine all the reactions in this problem. (25%)



(五) 機械製造

9. 球形研磨輪（半徑  $R$ ，見圖 1）是自由曲面的加工利器。但下列問題是製造業者該注意之事。請以直角座標系計算下列問題：
- (1) 該球形研磨輪的球表面如何以  $\psi$  與  $\beta$  表示？ (5%)
  - (2) 該球形研磨輪沿著  $\psi$  方向與沿著  $\beta$  方向的單位切線如何表示？ (5%)
  - (3) 該球形研磨輪的單位法線是同時垂直於(2)(3)之兩單位切線，請計算該球形研磨輪的單位法線？ (5%)
  - (4) 該球形研磨輪在  $\psi = 90^\circ$  的單位法線是否可從(3)的結果導出？ (2%) 為什麼？ (3%)
  - (5) 若研磨輪轉速為  $N \text{ rpm}$ ，且不考慮進給，則該球形研磨輪的球表面的線速度表示式 = ? (5%)

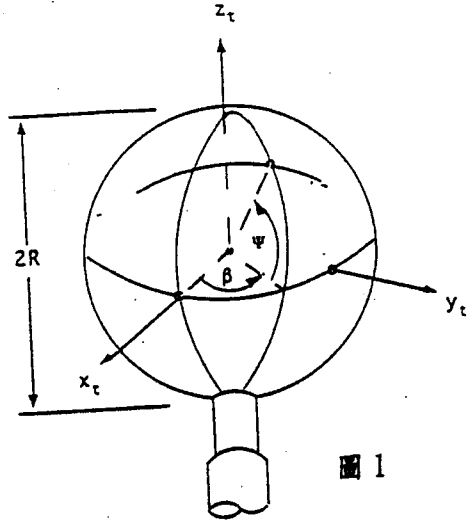


圖 1

10. 如圖 2 所示乃是拉伸(drawing)圓柱，使斷面積變小成為線材。圖中是圓薄片所受之力， $\mu$  是摩擦係數， $D$  是在  $x$  處的直徑， $p$ 、 $\sigma_x$  是垂直應力。

- (1) 在  $x$  處的斷面積是多少？在  $x$  處的拉力是多少？(5%)
- (2) 在  $x+dx$  處的斷面積是多少？在  $x+dx$  處的拉力是多少？(5%)
- (3) 圓薄片與模具管壁交集的面積是多少？圓薄片受模具管壁之垂直力，且沿  $x$  軸方向是多少？(5%)
- (4) 圓薄片受模具管壁之摩擦力，且沿  $x$  軸方向是多少？(5%)
- (5) 從靜力學觀點，列出軸向力的平衡方程式。(5%)

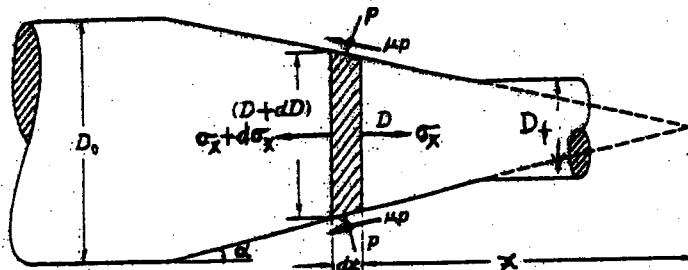
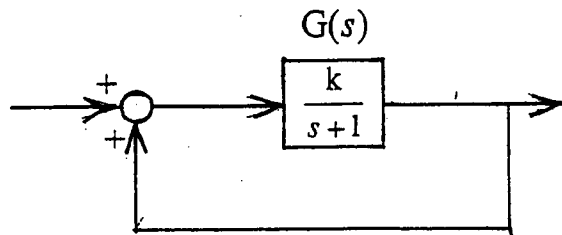


圖 2

(六) 自動控制

- (25%) II. a) Sketch the Nyquist-locus of  $G(s) = k/(1+s)$  where  $k$  is a positive gain. (15%)
- b) Apply the Nyquist theorem for the positive feedback system as shown, and determine the stability condition for  $k$ . (10%)



(背面仍有題目, 請繼續作答)

(25%) 12. Consider the closed-loop system shown below.

a) Draw a root locus diagram. (10%)

b) What value of gain  $K$  will give the damping ratio of the closed-loop poles equal to 0.6?

Locate these closed-loop poles on the root locus diagram. (15%)

