

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一. Linear Algebra (50%)

1. Matrix calculation.

(a) Given $\mathbf{A} = \begin{bmatrix} 1 & 5 & 12 \\ 1 & 5 & -2 \\ 1 & -4 & 2 \end{bmatrix}$. Find the Gram-Schmidt QR factorization of \mathbf{A} . (10%)

(b) Given $\mathbf{A} = \begin{bmatrix} 10 & 5 \\ -11 & 2 \\ -2 & 14 \end{bmatrix}$. Find a singular value decomposition of \mathbf{A} . (10%)

2. Let $\mathbf{A} = (a_{ij})$ be an $n \times n$ matrix with eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$.

(a) Show $\prod_{i=1}^n \lambda_i = \det(\mathbf{A})$. (5%)

(b) Show $\sum_{i=1}^n \lambda_i = \sum_{i=1}^n a_{ii}$. (5%)

(c) Suppose \mathbf{A}_r is the matrix formed by deleting the last $n - r$ rows and columns of \mathbf{A} . And we also assume \mathbf{A} is symmetric and $\lambda_i > 0$ for $i = 1, \dots, n$. Show $\det(\mathbf{A}_r) > 0$, $r = 1, \dots, n$. (10%)

3. Find the curve $y = C(-1)^x + D(2)^x$, which gives the least squares fit to points $(x, y) = (0, 0), (1, 4), (2, 6)$. (10%)

二. Discrete Mathematics (50%)

4. (20%) Don't just write down the answer without explanations.

(a) Determine the number of paths in the xy -plane from (m, n) to (p, q) , $m, n, p, q \in$ positive integer or zero, $m < p$, $n < q$, where each such path is made up of individual steps going one unit to the right $(x, y) \rightarrow (x+1, y)$ or one unit upward $(x, y) \rightarrow (x, y+1)$. (5%)

(b) If $(m, n) = (0, 0)$, $(p, q) = (7, 4)$, how many of the paths in part (a) do not use the path from $(2, 2)$ to $(3, 2)$ to $(4, 2)$ to $(4, 3)$? (5%)

(c) If $(m, n) = (0, 0)$, $(p, q) = (7, 4)$, how many of the paths in part (a) do not pass through the points $(0, 1)$, $(1, 2)$, $(2, 3)$, $(3, 4)$? (5%)

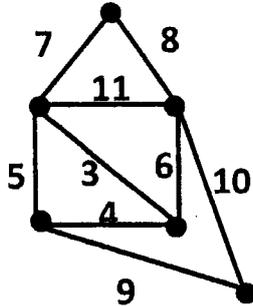
(d) If an additional type of move $(x, y) \rightarrow (x+1, y+1)$ is allowed, how many of the paths in part (a) if $(m, n) = (0, 0)$, $(p, q) = (7, 4)$? (5%)

(背面仍有題目,請繼續作答)

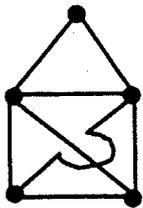
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5. (20%) Don't just write down the answer without explanations.

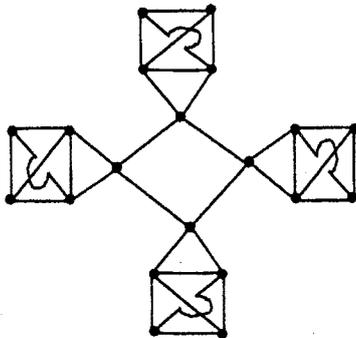
(a) If the cost of each edge is given, determine the cost of the minimum spanning tree in the following figure? (5%)



(b) How many different spanning trees in the following figure?(5%)



(c) How many different spanning trees in the following figure? (10%)



6. (10%) Find a formula for the convolution of each of the following pairs of sequences where n belongs to integers.

(a) $a_n = 1, 0 \leq n \leq 5, a_n = 0, \text{ for all } n \geq 6;$

$b_n = n, \text{ for all } n \geq 1$ (5%)

(b) $a_n = (-1)^n, b_n = (-1)^n, \text{ for all } n \geq 1$ (5%)