系所組別：全校

考試科目：微積分

考試日期：0708，節次：3

※ 考生請注意：本試題不可使用計算機

1. Define $f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

   a) Find f'(0) (6%)

   b) Is f'(x) continuous at x=0 ? (6%)

2. Evaluate $\int_0^1 x^2 \cdot \ln(x^3 + 1)dx$ (10%)

3. Show that $\frac{\sin^{-1}x}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{2^{2n}(n!)^2}{(2n+1)!} x^{2n+1}$, $\forall x: |x| < 1$ (12%)

4. The cardioid $r = 2(1 + \cos\theta)$ is rotated about the polar axis **y=0.**
   Find the area of the surface generated. (12%)

5. Evaluate $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1} dydx$ (10%)

6. Convert $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)dydx$ to polar coordinates and evaluate. (10%)

7. Find $\frac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$ if $w = x^2 + y^2 + z^2$
   and $z^3 - xy + yz + y^3 = 1$ (10%)

8. Two sides of a triangle are 10^{cm} and 15^{cm}, and are increasing at 3 $^{cm}/_{sec}$ and 4 $^{cm}/_{sec}$, respectively, which the included angle is $^{\pi}/_{3}$ and decreasing at 0.5 $^{rad}/_{sec}$. Is the third side increasing or decreasing ? at what rate ? (12%)

9. Suppose the utility of purchases of x, y, z units of three different kinds of product is given by $u = 5x^{\frac{1}{3}}y^{\frac{2}{3}}z^{\frac{1}{2}}$, where the price per unit of the products is \$2, \$5, and \$1, respectively. If a consumer has \$90 to spend, how many units of each product should be purchased to achieve maximum utility ? (12%)