

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (18 points) Find the following limits :

(a) (6 points) $\lim_{x \rightarrow \infty} \sqrt{x+3} \cdot (\sqrt{x+2} - \sqrt{x+1})$

(b) (6 points) $\lim_{x \rightarrow 0} \frac{x - \sin x}{2 \tan(\frac{x}{2}) - x}$

(c) (6 points) $\lim_{n \rightarrow \infty} (\frac{n}{n^2+1} + \frac{n}{n^2+4} + \dots + \frac{n}{2n^2})$

2. (10 points) Let $f(x) = \sin^{-1} \frac{x}{2} + \sqrt{4-x^2}$, for each $x \in [-2, 2]$

(a) (5 points) Find $f'(x)$

(b) (5 points) Find extreme values of $f(x)$

3. (12 points) Evaluate the following integrals :

(a) (6 points) $\int_0^{\frac{\pi}{2}} \sin 2x \cdot \cos^2 x \, dx$

(b) (6 points) $\int_0^1 \frac{dx}{e^x + 1}$

4. (10 points) Find the convergence set of $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n \cdot 4^n}$.

5. (10 points) Gauss' probability integral is $I_0(\alpha) = \int_0^{\infty} e^{-\alpha x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}$

Evaluate $I_1(\alpha) = \int_0^{\infty} x e^{-\alpha x^2} dx$ and $I_2(\alpha) = \int_0^{\infty} x^2 e^{-\alpha x^2} dx$.

6. (10 points) Determine and classify the stationary points of the function $f(x, y) = xy + (x - y)^3$.

7. (10 points) Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sin(x^3) \, dx dy$.

8. (10 points) Evaluate $\iint_R xy \, dA$, where R is the region bounded by $y = \sqrt{2x - x^2}$ and $y = 0$, by converting to Polar Coordinates.

9. (10 points) A certain gas satisfies the law $pV = T - \frac{4p}{T^2}$ where p=presure, V=volume, and T=temperature. Calculate $\frac{\partial T}{\partial p}$ and $\frac{\partial T}{\partial V}$ at the point where $p = V = 1$ and $T = 2$.