

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10 %) Find the limit: $\lim_{x \rightarrow 0} \frac{x^2 \sin(1/x)}{\tan x}$.

2. (10 %) Find the limit: $\lim_{x \rightarrow \infty} [x - x^2 \ln(1 + \frac{1}{x})]$.

3. (12 %) Give $\ln(x^2 + y^2) + 2 \cdot \arctan \frac{x}{y} = 0$, find the values of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when $x = 0$ and $y = 1$.

4. (10 %) Evaluate $\int_0^{\frac{\pi}{2}} \cos x \sqrt{1 - \cos x} dx$.

5. (10 %) Evaluate $\int x \cdot \ln(1 + x^2) dx$.

6. (12 %) Evaluate $\int_0^2 \int_x^2 x \sqrt{1 + y^3} dy dx$.

7. (12 %) Use polar coordinates to combine the sum

$$\int_{\sqrt{2}}^2 \int_{\sqrt{4-x^2}}^x x^{-1} y^2 dy dx + \int_2^{2\sqrt{2}} \int_0^x x^{-1} y^2 dy dx + \int_{2\sqrt{2}}^4 \int_0^{\sqrt{16-x^2}} x^{-1} y^2 dy dx$$

into one double integral. Then evaluate the double integral.

8. (12 %) Show that $\sum_{n=2}^{\infty} \ln(1 - \frac{1}{n^2})$ converges and finds its sum.

9. (12 %) Find the local maximum and minimum values and saddle point(s) of the function

$$f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$$

(本試題結束)