編號： 19

國立成功大學 103 學年度轉學生招生考試試題

系所組別：統計學系

考試科目：統計學

考試日期：0713，節次：4

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A statistics instructor would like to predict students' final exam scores in terms of their midterm exam scores. She considers a simple regression model to fit the previous data. Let y_i be the final score and x_i the midterm score for the ith student. Based on the previous data, she obtained some simple descriptive statistics. The means of final and midterm scores are 60 and 76 points, respectively. Moreover, the standard deviation of the midterm and final scores were 8 and 10 points, respectively. A simple regression model she used is

$$y = \beta_0 + \beta_1 x + \varepsilon, \qquad \varepsilon \sim N(0, \sigma^2).$$

After she fit the regression model to the previous data, she obtained

$$\sum_{i=1}^{100}(x_i - \bar{x})(y_i - \bar{y}) = 6{,}000; \qquad \sum_{i=1}^{100}(x_i - \bar{x})^2 = 10{,}000.$$

and the AVOVA table as follows.

Source	df	Sum of Squares	Mean Squares	F value	*p*-value
Regression	1	7,680	7,680	174.2	less than 0.0001
Residual	98	4,320	44.08		
Total	100	12,000			

a. Determine the estimated regression equation. (5%)

b. Test to determine whether there is evidence of a linear relationship between the final scores and the midterm scores. (5%)

c. If Meng-Yu scored a 90 on the midterm, what did the instructor predict for her score on the final? (5%)

d. Johnny in the class scored 95 on the midterm but got overconfident and slacked off so that he only got a 30 score. How big is his residual? (5%)

e. Develop a 95% confidence interval to predict the final score for a 70 on the midterm. (5%)

f. Based on the given information, could you estimate a student's midterm score from the final score? Explain. (5%)

2. There are two students, Shao-Wei and Bo-Ren, in a statistical class. They both love to watch baseball games, but their favorite teams are different. Shao-Wei likes St. Louis Cardinals but Bo-Ren likes Boston Red Sox. Suppose that both their favorite teams went to World Series and have an equal chance of winning the championship (best of seven). However, past records indicate that when Cardinals wins the championship, it wins the first game of the series 80% of the time. When Cardinals loses the series, it wins the first game 20% of the time. The first game is over; Boston has won. What is the probability that Boston will win the series? (10%)

3. Assume ***X*** and ***Y*** are two continuous random variables. Suppose that Var(**X**)=4, Var(**Y**)=2, and Cov(**X**, **Y**) = 1. What is Var(**Y**-**X**) ? (5%)

國立成功大學 103 學年度轉學生招生考試試題

系所組別：統計學系

考試科目：統計學

考試日期：0713，節次：4

4. A handsome and charming college basketball player, Ting-Yi historically made 40% of his free throws. Suppose one afternoon his basketball coach, Yi-Xiu, asked him to shoot 10 free throws. Let **X** be a random variable representing the number that Ting-Yi made the shots of these 10 free throws. Suppose each throw is independent.

a. What is the probability distribution of **X**? (5%)

b. What are the mean and variance of **X**? (5%)

c. Yi-Xiu, a well-known hard coach, always asks Ting-Yi to practice 500 free throws, but fortunately since Yi-Xiu did very well in his statistical test that morning, Yi-Xiu only asked Ting-Yi to shoot 100 free throws. What is the approximate probability that Ting-Yi would make at least 50 shots of these 100 free throws? (5%)

d. During these 100 free shots, they recorded the output as follows:

	make the shots	miss the shots
Frequency	45	55

Yu-Jing, the manager of the basketball team, claims Ting-Yi did better than before. State the hypotheses that can be used to test Yu-Jing's claim at a 0.05 level of significance. What is your conclusion? (10%)

5. Suppose the number of cars that arrive at a car wash during one hour is described by a Poisson probability distribution with a mean of 10 cars per hour. Now we are interested in the time, denoted by **X**, between the arrivals.

a. What is the probability distribution of **X**? (5%)

b. What is the probability that arrival time is greater than 6 minutes? (5%)

6. A random sample of second-year university students who enrolled in a business statistics course was drawn at the course's completion. These students were asked how many total hours he or she spent doing homework for this course this semester. The data are listed here.

Yan-Ling	Yan-Jia	Wei-Ting	Ting-Rong	Meng-Yu
10	48	37	35	32
Quan-Jin	Bo-Kai	Bo-Ren	Shao-Wei	Yan-Hua
48	49	36	29	46

The mean and standard deviation for this data are 37 and 12, respectively. It is assumed that the data is from a normal distribution.

a. What is the 90% confidence interval estimate of population mean total hours that students spent doing homework for this course this semester? (5%)

b. The instructor of this course has recommended that students devote 3 hour to do homework per week for the duration of the 16-week semester, for a total of 48 hours. Test to determine whether there is evidence that students spent less than the recommended amount of time at a 0.05 level of significance. (10%)

c. Do you think this sample is good to test the hypotheses? Explain. (5%)

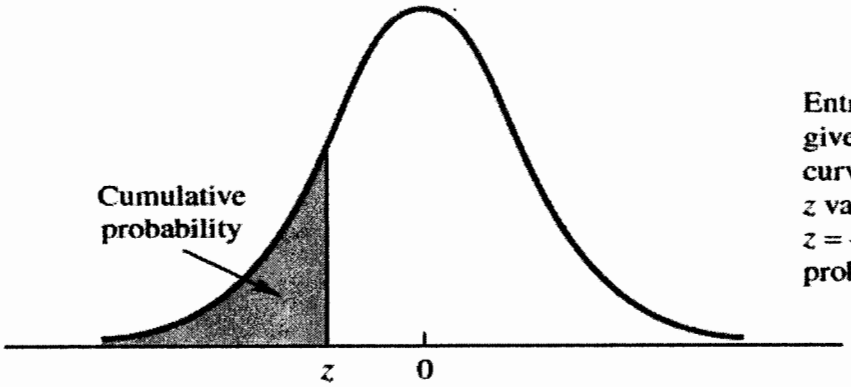
國立成功大學 103 學年度轉學生招生考試試題

系所組別：統計學系

考試科目：統計學

考試日期：0713，節次：4

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641