1. When you visit the Main Library of NCKU on a given day, the probability that someone is reading the current issue of China Times is 0.3, the probability that someone is reading the United Days is 0.35 and the probability that at least one of the above two newspapers is being read by someone is 0.5. What is the probability that
(a). Both newspapers are being read. (6 points)
(b). Neither of the two is being read. (7 points)
(c). Exactly one is being read. (7 points)

2. Starting at a fixed time, each car entering (independently) the intersection of University Road and Victory Street is observed to see whether it turns left, right or goes straight head. The experimenter terminates as soon as a car is observed to turn left. Let X be the number of cars observed and assume that the probability that a randomly chosen car turns left is 0.4.
(a). What is the probability mass function? (6 points)
(b). Find $P(X > 3)$. (7 points)
(c). Find the expected number of cars observed. (7 points)

3. The Pareto distribution is frequently used as a model in study of incomes. Its density function is given by $f(x; \theta) = \theta \frac{a^\theta}{x^{\theta+1}}$, for $x > a$, $a > 0$, where a is a known constant and θ is an unknown parameter, $\theta > 1$. Based on a random sample $X_1, X_2, \ldots, X_n$ from the pareto distribution, the estimates of θ and E(X) are desired.
(a). Give the likelihood function $L(\theta)$. (6 points)

(b). Show that the expected value of X is $\frac{\theta}{\theta-1}a$. (8 points)

(c). Show that the M.L.E. of θ is $\hat{\theta} = \frac{n}{\sum_{i=1}^{n} L_n(x_i/a)}$, where L_n stands for natural logarithm. Make sure that you show all the necessary steps. (8 points)
(d). Assume now that a = 30. We are interested in estimating the population mean income. Our estimator will be based on the following 10 incomes:

25 40 39 57 12 70 37 80 61 27

The data above are given in the unit of NT$10,000 and represents a random sample from the whole population of incomes. Give the maximum likelihood estimate of the population mean income. (8 points)

4. There are various noise levels at the Watzit plant and the operations supervisor, C. S. Wang, wishes to check the time required for workers to assemble a watzit under the varying conditions. The noise level X, measured in decibels (dB), and the time required for assembly Y, measured in mimutes, for eight workers is given in the following:

X:	10	10	30	30	50	50	70	70
Y:	11	14	14	18	18	19	24	22

(a). Do you think a simple linear regression model would be appropriate to explain the time required for assembly in terms of the noise levels? Justify your answer. (6 points)

Assume, for the rest of the problem, that X and Y are related according to the simple linear regression model.
(b). Calculate the least squares estimates of the slope and y intercept of the population regression line. (6 points)
(c). Give the equation of the estimated regression line and use it to predict the average time required for assembly for a randomly chosen worker if the noise level at the watzit is 30 dB. (6 points)
(d). Suppose that Mr. C. S. Wang had believed, prior to seeing these data, that β_1 (the slope) = 0.21. Does the data support his prior belief at $\alpha = 0.05$? Justify your answer. (6 points)
(e). Compute a 95% confidence interval for the expected time required for the assembly when the noise level is 45 dB. (6 points)

NOTE: $t_6(0.95) = 1.943$ $t_6(0.975) = 2.447$ $t_7(0.95) = 1.895$
$t_7(0.975) = 2.365$ $t_8(0.95) = 1.860$ $t_8(0.975) = 2.306$