臺灣綜合大學系統 111 學年度學士班轉學生聯合招生考試試題

科目名稱	微積分A	類組代碼	共同考科
		科目碼	E0011

※本項考試依簡章規定所有考科均「不可」使用計算機。　本科試題共計 1 頁

1. (10 points) For what values of a and b, $a, b \neq 0$, is the following equation true?

$$\lim_{x\to 0}\left(\frac{e^{ax}-1}{x^2} - ax + \frac{b}{x}\right) = \frac{1}{2}$$

2. (a) (5 points) Let $f(x) = \sec^2 x$ on $(0, \frac{\pi}{2})$ and f^{-1} be the inverse function of f. Find $(f^{-1})'(4) =$____________.

   (b) (5 points) Let $f = f(x, y)$ be a differentiable function of x and y, and let $x = rs, y = r + s$ and $h(r, s) = f(x, y) = f(rs, r + s)$. Assume $\frac{\partial f}{\partial x}(1, 2) = 2$, $\frac{\partial f}{\partial y}(1, 2) = 1$. Find $\frac{\partial h}{\partial s}(1, 1) =$____________.

3. (10 points) If $f(x) = \int_0^{x^2}(1 - t^2)e^{t^2}\,dt$, on what interval(s) is f increasing?

4. (10 points) Let $a > 0$ be a constant. Evaluate

$$\int_0^a \frac{x^2}{(x^2 + a^2)^{3/2}}dx.$$

5. (10 points) Find the area of the region that lies inside the curve $r = 4\sin\theta$ and outside the curve $r = 2$.

6. (10 points) For what real values of p does the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$ converges?

7. (10 points) Let $f(x)$ be the function defined by the power series

$$\sum_{n=0}^{\infty} \frac{(x+2)^{2n}}{(n+3)!}.$$

   Try to express $f(x)$ as an elementary function.

8. (10 points) Find the global maximum and global minimum of $f(x, y, z) = x^2 + y^2 + z^2$ on the surface $x^2 + 2y^2 + 3z^2 = 1$ by using the method of Lagrange multipliers.

9. (10 points) Evaluate $\iint_R \frac{y}{x} e^{xy}\, dA$, where R is the region in the first quadrant bounded by lines $y = x$, $y = 3x$, and the hyperbolas $xy = 1$, $xy = 3$.

10. (10 points) Evaluate the line integral $\int_C (1 - y^3)dx + (x^3 + e^{-y^2})dy$, where C is the arc of the the circle $x^2 + y^2 = 4$ traversed counter-clockwise from $(2, 0)$ to $(-2, 0)$.