

編號： 32 系所：全校系所

科目：微積分

本試題是否可以使用計算機： 可使用， 不可使用（請命題老師勾選）

1. Compute the following limit:

10% (a) $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \sqrt{\left(\frac{3i}{n}\right)^7 + \left(\frac{3i}{n}\right)^4}$.

5% (b) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$.

2. Let $f(x) = \int_1^{e^x \ln(x+e)} \frac{1}{\sqrt{x^4 + x^2 + 5}} dx$ defined on $(-1, \infty)$.

5% (a) Show that f is a strictly increasing function.

5% (b) Find $(f^{-1})'(0)$.

10% 3. Compute the improper integral $\int_0^1 x^{-2/3} \ln x dx$.

10% 4. Find the radius of convergence of the infinite series $\sum_{n=0}^{\infty} \left(\frac{n}{2n+1}\right)^n x^n$.

10% 5. Let f be a real valued function defined on \mathbb{R} with $f''(x) > 0$ for all x . Show that

$$f(x) \geq f'(0)x + f(0) \quad \text{for all } x.$$

10% 6. Compute the iterated integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2 + y^2) dy dx$.

10% 7. Find the local extreme values of the function

$$f(x, y) = xy + \frac{1}{x} + \frac{8}{y}$$

for $xy \neq 0$.

10% 8. Use the method of Lagrange Multiplier to find the maximum value of the function

$$f(x, y, z) = x + 3y - 2z$$

defined on \mathbb{R}^3 subject to the constraint $x^2 + y^2 + z^2 = 14$.

15% 9. Find the area of the region Ω bounded by the curves $x^2 - 4xy + 4y^2 - 2x - y - 1 = 0$ and $y = \frac{2}{5}$.