※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. Compute the following limits, if exist :　　　　　(各 7%)

   a) $\lim_{x \to 0} \frac{\sin x}{x - [x]}$, where $[x]$ is the Gauss function,

   b) $\lim_{x \to 1} \frac{\int_1^{\sqrt{x}} e^{-t^2} dt}{\ln x}$.

2. Find the local extrema of $f(x) = x^2 \ln x$ for $x > 0$, discuss concavity and find the point of inflection.　　(12%)

3. Calculate the following integrals :　　　　　　　(各 7%)

   a) $\int_2^4 \frac{dx}{x \ln \sqrt{x}}$

   b) $\int_0^1 x \cdot \arctan x^2 \, dx$

4. Find the area of the surface generated by revolving the curve $6xy = x^4 + 3$ from $x = 1$ to $x = 3$ about the x-axis.　　(12%)

5. Use $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$, prove $\sum_{n=1}^{\infty} \frac{n^2 x^n}{n!} = (x^2 + x)e^x$,

   and find the sum $\sum_{n=1}^{\infty} \frac{n^3}{n!}$.　　　　　　　(12%)

6. Find the average value of $f(x,y) = xy$ over the quarter circle $x^2 + y^2 \le 1$ in the first quadrant.　　(12%)

7. Evaluate $F(x,y) = \int_0^{\infty} \frac{e^{-xt} - e^{-yt}}{t} \, dt$ for $x > 0, y > 0$.　　(12%)

8. Find the points on the curve $17x^2 + 12xy + 8y^2 = 100$ that are closest to and farthest away from the origin.　　(12%)