臺灣綜合大學系統

106 學年度

轉學生聯合招生考試

試　題

類組：A06/A07/A09/A10/

A11/B12/B17

科目名稱：微積分 A

科目代碼：E0011

臺灣綜合大學系統106學年度學士班轉學生聯合招生考試試題

科目名稱	微積分 A	類組代碼	
		科目碼	E0011

※本項考試依簡章規定各考科均「不可以」使用計算機　　本科試題共計 2 頁

1. (10 points) Given a curve C in $\mathbb{R}^2$ defined by

$$\ln(1-x^3+y^3)-4=0.$$

Find the point on C at which the tangent line is vertical.

2. (10 points) If the function

$$f(x)=\begin{cases}\frac{\sin(4x)+a-2b}{3x} & x\neq 0\\ 2a+b & x=0\end{cases}$$

is continuous at $x=0$, then $(a,b)=?$

3. (10 points) Write down the first three terms (three lowest order terms) of the Taylor series of $\frac{\tan^{-1}(2x)}{1-x}$ at 0. (Hint: $\tan^{-1}u=\int ?du$)

4. (10 points) Evaluate the following integral:

$$\int_0^1\int_{\sqrt{x}}^1 x\,\cos(y^5+2)\;dy\,dx.$$

5. (10 points) From the equation

$$e^{x^2}+y^2\sin(2x)=4y,$$

Solve $\frac{dy}{dx}$ in terms of x and y

6. (10 points) Find all values of a so that the series

$$\sum_{n=1}^{\infty}\sin\left(\frac{1}{n^{3a-1}+3}\right)$$

is *divergent.*

背面有題，請繼續作答。

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7. (10 points) Compute the following improper integral

$$\int_0^\infty e^{-4x^2}\,dx.$$

8. (10 points) Compute the line integral

$$\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}},$$

where

$$\vec{\mathbf{F}}(x,y) = (4y + 6ye^{2x}, 6x + 3e^{2x})$$

and C is the closed loop formed by traveling from $(-2,0)$ to $(4,0)$ to $(3,3)$ to $(-1,3)$ and back to $(-2,0)$ by straight lines.

9. (10 points) Given the function $F : \mathbb{R}^3 \to \mathbb{R}$ by

$$F(x,y,z) = e^{x+y^2+\cos z}.$$

At $(0,0,0)$, find the direction along which the function *decreases* most rapidly and find the corresponding rate of change.

10. (10 points) A rectangular box is formed by cutting four equal corners from a square of side 3 and then folding up (see the figure below). Find the maximum possible volume of the box.